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Stochastic Models in Insurance, Risk and Pension Funds[1]

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Abstract

We provide some results on stochastic modelling in the insurance and pension funds. We discuss several well-known insurance premium principles and offers some alternatives for premium calculations. We provide the feasibility ratio when death time follows the gamma distribution. We also discuss the testing of the scale hypothesis of the gamma distribution and provide the illustrative example analyzing the US markets loss amounts in USD, which occurred in 1990 and 1999, obtained from Property Claim Service.

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1. INTRODUCTION

Although methods for calculation of insurance premiums and policy reserves may be different, most of them are derived from one and the same probability inequality. All methods use a risk function (e.g. a loss or profit function). The paper presents several well-known insurance premium principles and offers some alternatives for premium calculations. It does not consider the deterministic model which avoids randomness of insurance events and corresponding benefits by using the fictitious population principle (see [Sekerová and Bilíková 00]). Instead a stochastic model is used that describes cash-flow of a policy product as a random variable where contingency may also apply to interest rates. In the first part life and non-life insurance are discussed, while the second part deals with pension schemes (see [Škrovánková 04] for details).

2. METHODS FOR CALCULATING PREMIUMS

THEOREM 1. **Markov inequality** *Let X be a random variable $a, r \geq 0$. Then*

$$P\{|X| \geq a\} \leq \frac{E|X|^r}{a^r} \tag{1}$$

The inequality can be generalized as follows.

THEOREM 2. *Let X be a random variable, $a \in R$. Let $\Phi(x, y)$ be any Lebesgue measurable bivariate function and $\nu(x)$ any non-negative and non-decreasing func-*

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tion such that $E[\nu(X)] < \infty$ *and* $E\left[\Phi(X,y)\nu(X)\right] < \infty$ *for all relevant* y. *Then*

$$P\{X \geq a\} \leq \frac{E\left[\Phi(X,a)\nu(X)\right]}{E[\nu(X)]} \tag{2}$$

Supposing $X \geq 0$ with probability 1, $a \geq 0$, $\Phi(x,y) = x^r/y^r$ and $\nu = 1$ in Markov inequality, one gets (1) as a special case of (2).

DEFINITION 1. *By a risk measure* π *we understand a mapping from the set of risk (random) variables to the set of real numbers.*

In what follows we put $X = S$, where S is a risk variable. Using (2) it is not difficult to prove that, under certain conditions, for some $\alpha, 0 \leq \alpha \leq 1$ there exists a minimal value π_M such that

$$P[S > \pi_M] \leq E[\Phi(S,\pi_M)\nu(S)]/E[\nu(S)] \leq \alpha \leq 1.$$

This value is the solution of the equation

$$E[\Phi(S,\pi_M)\nu(S)]/E[\nu(S)] = \alpha \tag{3}$$

and is called a *Markovian risk measure of the risk variable* S *at level* α. When $\alpha = 1$, the equation

$$E[\Phi(S,\pi_M)\nu(S)]/E[\nu(S)] = 1 \tag{4}$$

is called the *unifying equation.* Many well-known insurance premium principles and corresponding risk measures follow from (4) as special cases.

—**the mean value principle** Let S be a risk variable. For a given non-decreasing and non-negative function f such that $E\left[f(S)\right] < \infty$, the mean value risk measure is the solution of the equation $f(\pi) = E\left[f(S)\right]$. It can be obtained from (4) if one puts $\Phi(s,p) = f(s)/f(p)$ and $\nu = 1$.

—**the zero-utility premium principle** Let u be a non-decreasing utility function. The zero-utility risk measure is the solution of the equation $u(0) = E(u(\pi - S))$. This is also a Markovian risk measure, if one chooses $\Phi(s,\pi) = u(\pi - S)/u(0)$ and $\nu = 1$.

—**the Swiss premium calculation principle** This principle generalizes the two preceding principles. Let w be a non-decreasing and non-negative function and $0 \leq z \leq 1$ be a parameter. The Swiss premium calculation principle is the solution of $E(w(S - z\pi)) = w((1-z)\pi)$. It suffices to put $\Phi(s,\pi) = w(s - z\pi))/w((1-z)\pi)$ and $\nu = 1$, with z either 0 or 1. These are classical principles. However, more general risk measures can be derived from (4) (see, e.g. [Goovaerts et al. 03]).

3. THE CONSTANT INTEREST RATE CASE

As mentioned in [Potocký 96], one usually calculates premium under the requirement that the mean value of the present value of profit of an insurance office be zero. Using the zero-utility premium principle and supposing a linear utility function of the insurance (life) office such that $u(0) = 0$, we obtain a well-known relation $\pi = E(S)$, where S is the present value of the claims of an insured and π is a single premium. This result is easily extended to the case of annual premiums.

Let b_j be a claim payment at the end of j-th year, P_{j-1} be an annual premium at the beginning of the j-th year, $j = 1, 2, \ldots$ Then the present value of the profit to the life office is

$$Z = \sum_{j=0}^{K} P_j v^j - b_{K+1} v^{K+1}, \tag{5}$$

where K (more accurately K_x) means the curtate future lifetime of a person aged $x, v = (1+i)^{-1}, i$ is an annual interest rate. For its distribution we have $P(K_x = k) =_k |q_x$ (the corresponding distribution function is $F(k) =_{k+1} |q_x$), where ${}_k|q_x$ means the probability of a person aged x to survive to age $x+k$ but not to age $x+k+1$. Similarly ${}_j p_x$ is the probability of a person aged x to survive the next j years. The expected value of the profit is

$$E(Z) = \sum_{h=0}^{\infty} P_h v^h \ {}_h p_x - \sum_{j=0}^{\infty} b_{j+1} v^{j+1} \ {}_j|q_x. \tag{6}$$

An alternative approach for some insurance products is to consider risk at the time of the last benefit payment. It means to calculate the accumulated value of the profit and then compare its expectation to zero.

Consider a classical n-year endowment assurance with a premium paid annually during n years, the sum assured $1Sk$ and the benefit payment at the end of the policy year of death of the insured or at age $x+n$ if the insured has not previously died. Then the present value of the profit to the insurance office is $Z = PV - U$, where P is the annual premium,

$$V = \begin{cases} \ddot{\mathrm{a}}_{\overline{\mathrm{K+1}}|} & \text{for } K = 0, 1, \ldots, n-1, \\ \ddot{\mathrm{a}}_{\overline{\mathrm{n}}|} & \text{for } K = n, \ldots \end{cases}$$

$$U = \begin{cases} v^{K+1} & \text{for } K = 0, 1, \ldots, n-1, \\ v^n & \text{for } K = n, \ldots, \end{cases}$$

where U is the present value of the benefit payment $1Sk$, which is a special case of (5). The definitions of $\ddot{\mathrm{a}}_{\overline{\mathrm{K+1}}|}, \ddot{\mathrm{a}}_{\mathrm{x};\overline{\mathrm{n}}|}$ and other actuarial symbols are e.g. in [Sekerová and Bilíková 00]. We have

$$E(U) = \sum_{0}^{n-1} v^{k+1} P(K=k) + \sum_{n}^{\infty} v^n P(K=k) = A^1_{x;\overline{n}|} + v^n \ {}_n p_x = A_{x;\overline{n}|}.$$

Similarly $V = (1-U)/d$ and thus $E(V) = (1 - E(U))/d = \ddot{\mathrm{a}}_{\mathrm{x};\overline{\mathrm{n}}|}$. Putting $E(Z) = 0$ we have a known result, namely

$$P = E(V)/E(U) = A_{x;\overline{n}|}/\ddot{\mathrm{a}}_{\mathrm{x};\overline{\mathrm{n}}|} \tag{7}$$

It can be rewritten to the form

$$P = d/(1/A_{x;\overline{n}|} - 1). \tag{8}$$

Consider now the value of the profit to the insurance office at the time $\tau = \min(K+1, n)$, when the benefit is paid. We recall that τ is a random variable. We express the profit value at the time τ as follows $Z = P(1+i)s_{\overline{\tau}|} - 1$ or equivalently

$$Z = P(1+i)^{\tau}/d - (1 + P/d). \tag{9}$$

The formula (9) is recommended to calculate the expectation and variance of Z. We have

$$E(Z) = P/d\{\sum_{0}^{n-1}(1+i)^{k+1}P(K=k) + (1+i)^n\ {}_np_x\} - (1+P/d).$$

Putting $E(Z) = 0$, we obtain the premium P. It is immediate that

$$P = d/\{\sum_{0}^{n-1}(1+i)^{k+1}P(K=k) + (1+i)^n\ {}_np_x - 1\}. \tag{10}$$

Although (10) looks more complicated than (7), the complexity is almost the same when computers are used. The advantage of (10) is obvious when interest rates are supposed to change stochastically as discussed in the third part of the paper. It is necessary to say that this approach sometimes underestimates premiums. The differences vanish with the increasing number of policies since then the central limit theorem holds.

If a random variable Y is defined as $Y = (1+i)^{\tau}$, then $U = Y^{-1}$. On the other hand $E(U) = E(Y^{-1}) \neq E(Y)^{-1}$. If the equality were true, the premium calculated by (7) would be the same as the premium calculated by (10). In fact, they will be different.

Remark As all incomes and outgoes of the insurance product are valued at the time which differs from zero, this approach requires a different way of calculation of policy reserves.

4. STOCHASTIC INTEREST RATES

Consider an investment for n years. Divide time interval $(0,n)$ into subintervals of unit length $(0,1),(1,2),...,(n-1,n)$. Let the interest rate i_t in the interval $(t-1,t)$ be a random variable. For an n-year endowment with unit sum assured the present value of the profit to the insurance company will be

$$P\left(1+\sum_{j=1}^{K}\prod_{k=1}^{j}(1+i_k)^{-1}\right) - \prod_{k=1}^{K+1}(1+i_k)^{-1} - \text{costs}.$$

If no costs are supposed, the premium will be

$$P = \frac{E\left(\prod_{k=1}^{K+1}(1+i_k)^{-1}\right)}{1+E\left(\sum_{j=1}^{K}\prod_{k=1}^{j}(1+i_k)^{-1}\right)}. \tag{11}$$

On the other hand the accumulated value of a sequence of the above mentioned investments will be $S_n = \sum_{t=1}^{n}\prod_{m=t}^{n}(1+i_m)$ and its expectation $E(S_n) = (1+j)s_{\overline{n}|j}$ provided $E(i_t) = j$ for $t = 1,\dots,n$, under the additional assumption that any interest rate does not depend on the previous ones.

The proof is based on the fact that $S_n = (1+i_n)(1+S_{n-1})$, $n \geq 2$. Since S_{n-1} and i_n are independent by assumption, we have $E(S_n) = (1+j)(1+E(S_{n-1}))$. The rest of the proof is immediate. If we apply this result to the endowment considered in the previous paragraph, then provided i_k are independent on the random variable

K defined in the second part of the paper, the profit to the company at time $\tau = \min(K+1, n)$ will be

$$Z = \begin{cases} P\left(\sum_{t=1}^{k+1} \prod_{m=t}^{k+1}(1+i_m)\right) - 1 & \text{for } K = 0, 1, \ldots, n-1 \\ P\left(\sum_{t=1}^{n} \prod_{m=t}^{n}(1+i_m)\right) - 1 & \text{for } K = n, \ldots \end{cases} \tag{12}$$

For its expectation we have

$$E(Z) = (P/d)\{\sum_{k=0}^{n-1}(1+j)^{k+1}P(K=k) + (1+j)^n\ {}_np_x\} - (1+P/d). \tag{13}$$

If we put $E(Z) = 0$, then it suffices to know the expectation $E(i_t)$ to calculate premium. On the other hand $E(1+i_t)^{-1}$ must be known if the formula (11) is used.

If the distribution of i_t is supposed to be known an alternative approach how to calculate premiums is at hand. If, e.g. $1+i_t$ has a log-normal distribution with $E(i_t) = j$ and variance $D(i_t) = s^2$ for $t = 1, \ldots, n$, then the profit at $\tau = \min(K+1, n)$ can be expressed as follows $Z = P\prod_{m=1}^{\tau}(1+i_m) - 1$. The premium is calculated from the equality $P(Z < 0) = p$, where p is a predetermined small number, e.g. 0.05. We have

$$P(Z<0) = \sum_{k=0}^{n-1} \Phi(\frac{-\ln P - (k+1)m}{\sigma\sqrt{k+1}})P(K=k) + \Phi(\frac{-\ln P - nm}{\sigma\sqrt{n}})\ {}_np_x,$$

where m and σ^2 are found from the equations $1+j = \exp(m + 1/2\sigma^2)$ and $s^2 = \exp(2m+\sigma^2)(\exp\sigma^2 - 1)$.

A new look at this problem is presented in [Bühlmann 95]. Let X_k be a stochastic cash payment in the time $k, k = 0, 1, \ldots, n$, i.e. benefits minus premiums plus costs in k-th year. Suppose that $E(\sum_{k=0}^{n} X_k^2) < \infty$. One then considers the expectation $E\sum_{k=0}^{n}\varphi_k X_k$, where $\varphi_k > 0$ with probability 1 for $k = 0, 1, \ldots, n$, and $\varphi_0 = 1$, which generalizes the above mentioned expected present value of the profit (by putting $\varphi_k = v^k$). In this setting X_k are technical variables, φ_k financial variables (or stochastic discount functions). Sometimes φ_k are preferred in the form $\varphi_k = \prod_{j=1}^{k} Y_j$, where Y_j (year to year variables) makes a decomposition of the profit (or loss) of an insurance company into annual losses possible. Moreover annual losses form a martingale and hence they are uncorrelated (A possible model for Y_j is

$$Y_j = \varepsilon(1-Z_j) + \delta Z_j$$

where Z_j are stochastic weights.). Then the premium is obtained from the equation

$$E\sum_{k=0}^{n}\varphi_k X_k = 0.$$

5. THE PENSION FUND MODEL

Of particular interest to pension fund administration is optimal asset allocation and feasibility ratio value. It is by now rather popular to model the losses as the gamma and Weibull variable and estimate parameters by standard maximum likelihood estimation (MLE) (see e.g. [Hogg, Klugman 84]). The well-known Wilks results on χ^2-asymptotics of likelihood ratio became the standard tool for testing the scale of such losses. However, in small samples asymptotical test is oversized and one wants to know the reliability of provided test.

In this section we discuss some properties of the pension fund investing within a financial market model given by [Battocchio et al. 03]. Such setup considers a financial market where there exist n risky assets and one riskless asset paying a constant interest rate r with the dynamics of the form

$$dS = \mathrm{diag}(S)\left(\mu dt + \Sigma^T dW\right)$$

and

$$dG = Grdt,$$

where W is a k-dimensional Wiener process. We assume μ and Σ constant. Such a model employees the Black-Scholes option pricing model introduced in 1973, formalized and extended by Merton. Let T indicate the (deterministic) date at which the subscriber retires and then the dynamic budget constraint can be written as

$$dR = (Rr + w^T M + k)dt + w^T \Sigma^T dW,$$

where $M = \mu - r\mathbf{1}$, $w = \mathrm{diag}(S)\theta$, and $k = uI(t \leq T) - v(1 - I(t \leq T))$. Here $I(t \leq T)$ is indicator of the interval $[0, T]$ and $\mathbf{1}$ is a vector of 1s.

5.1 The feasibility condition

The constant level of the contribution and pension rates are assumed to be optimally chosen by the fund. We follow the [Battocchio et al. 03] and put

$$E_0^\tau \left[\int_0^\tau k(s)\exp(-rs)ds\right] = 0,$$

which can be transformed into equation $\frac{u}{v} = \frac{E_0^\tau\left[\int_0^\tau \exp(-rs)ds\right]}{E_0^\tau\left[\int_0^\tau I(s\leq T)\exp(-rs)ds\right]} - 1$. Finally we recall the Definition 1 from [Battocchio et al. 03]:

Definition 2. *A pair of contribution and pension rates (u, v), $u, v > 0$ is said to be feasible if*

$$\frac{u}{v} = \frac{1 - E_0^\tau\left[\exp(-r\tau)\right]}{1 - E_0^\tau\left[I_{\tau\leq T}\exp(-r\tau)\right] - \exp(-rT)P(\tau \geq T)} - 1,$$

where $I_{\tau \leq T}$ is the indicator for the event $\tau \leq T$.

5.2 The gamma distribution

In [Battocchio et al. 03] the death time τ is supposed to follow the Weibull distribution. In [Sárazová and Stehlík 04] we study the behavior of the pension investment allocation with the respect to retirement age varying and we suppose that the death time τ follows the exponential distribution. There we can seen, following the approach of [Battocchio et al. 03], that when the time distribution is exponential with the scale parameter β, and retirement horizon T is rather large, the feasibility ratio $\frac{u}{v}$ is decreasing with β. The interpretation of this supports the ideas of classical pension reform, especially concerning the capitalization. More precisely, we have

$$\frac{u}{v} = \frac{1}{1 - \exp\left(-(r+\beta)T\right)} - 1,$$

which is decreasing function of β for any fix $T > 0$. Such behavior is connected with the effect of the Weibull family hazard function.

In present paper we assume that τ follows the gamma distribution

$$f(\tau|\gamma) = \gamma^v \frac{\tau^{v-1}}{\Gamma(v)} e^{-\gamma\tau}, \text{ for } \tau > 0, \tag{14}$$

where $\gamma > 0$ is scale parameter and $v > 0$ is shape parameter. In the following theorem (see [Potocký and Stehlík 04]) we provide the feasibility ratio under the gamma distribution.

THEOREM 3. *Let τ follow the gamma distribution (1). Then the feasibility ratio has the form*

$$\frac{u}{v} = \frac{(\gamma+r)^v - \gamma^v}{(\gamma+r)^v - \gamma^v\left(1 - \frac{\Gamma(v,(\gamma+r)T)}{\Gamma(v)}\right) - (\gamma+r)^v \exp(-rT)\frac{\Gamma(v,\gamma T)}{\Gamma(v)}} - 1, \tag{15}$$

where $\Gamma(v,t) := \int_t^{+\infty} \exp(-s)s^{v-1}ds$ is an incomplete Gamma function.

6. TESTING THE GAMMA MODEL

In this subsection we discuss the efficient test of the scale parameter hypothesis. Such procedure enables us to justify the adequacy of the scale parameter and to compute the appropriate feasibility ratio according to (12). The test has many economic and financial applications. To illustrate this, we employ this procedure to analyze the scale parameter of the US markets losses waiting times modelled by the homogeneous Poisson process.

Our setup considers independent random variables $y_1, y_2, \ldots, y_N$ distributed according to the gamma densities (1). For more extensive discussion on the scale hypothesis testing in the gamma family see [Stehlík 03, b]. We focus ourselves on the exact LR tests of the scale hypothesis

$$H_0 : \gamma = \gamma_0 \text{ versus } H_1 : \gamma \neq \gamma_0 \tag{16}$$

in the scale-homogeneous gamma family (i.e. $\gamma_1 = \ldots = \gamma_N \stackrel{\text{def}}{=} \gamma$) with known shape parameter. The exact p-value of the LR test of the hypothesis (16) is computed from the formula $p = 1 - F_N(-2\ln\lambda)$ where

$$\lambda = \frac{\max_{\gamma\in\Gamma_0} \prod_{i=1}^N f(y_i|\gamma_i)}{\max_{\gamma\in\Gamma_1} \prod_{i=1}^N f(y_i|\gamma_i)}$$

is the likelihood ratio of the test of the hypothesis (16), $\Gamma_0 = \{\gamma_0\}$ and $\Gamma_1 = \mathbb{R}_+ \setminus \Gamma_0$. Function F_N is the exact cdf of the Wilks statistics $-2\ln\lambda$ of the LR test of the hypothesis (16) under the H_0 and has the form (see [Stehlík 03, a])

$$F_N(\rho) = I(\rho)\left[F_{vN}^{\Gamma}\left\{-vNW_{-1}(-e^{-1-\frac{\rho}{2vN}})\right\} - F_{vN}^{\Gamma}\left\{-vNW_0(-e^{-1-\frac{\rho}{2vN}})\right\}\right],$$

and the Wilks statistics has the form

$$2G_{vN}\left(\gamma_0 \sum_{i=1}^{N} y_i\right) - 2G_{vN}(vN),$$

where for $u > 0$ we introduce the function $G_u(x) = I(x)(x - u \ln x)$. Here (and further) symbol $I(x)$ stands for indicator of the set $\mathbb{R}_+$, $W_k,\ k = -1, 0$ is the k-th branch of the Lambert W function and F_u^{Γ} is the cdf of the gamma distribution with the shape parameter u and scale parameter 1. The Wilks statistics $-2\ln\lambda$ has under the H_0 asymptotically χ_1^2-distribution (see [Wilks 62]).

There are two major advantages of the exact tests over asymptotic ones. The first is exact level of significance (the asymptotical tests are oversized) and possibility to compute the exact p-values. The second advantage is that you can evaluate the exact power of the test. Since the exact LR tests of the scale in the gamma family are rather powerful, it provides a good tool for comparison with the less powerful asymptotical tests. For example, the exact LR test of the scale of the exponential distribution is asymptotically optimal in the sense of the Bahadur exact slopes. Furthermore, the exact LR tests of the scale in the gamma family are unbiased, uniformly most powerful (UUMP) tests (see [Lehmann 86] for details).

6.1 Illustrating example

In this example we will illustrate exact testing for the gamma loss following the empirical studies for the PCS data obtained from Property Claim Services. Here we plot the exact p-value of the LR test of the hypothesis (16) and discuss the power of such test. The data concerns the US markets loss amounts in USD, which occurred in 1990 and 1999 adjusted for inflation. Only natural perils which caused damages exceeding 5 million dollars were taken into consideration. Two largest losses in this period were caused by Hurricane Andrew (24 August 1992) and the Northridge Earthquake (the January 17, 1994, Northridge, CA Earthquake). The exponential, lognormal, Pareto, Burr and gamma distributions were fitted to the waiting time data and the goodness-of-fit was tested. The χ^2 test accepted the exponential distribution with $\gamma = 30.97$. Let us consider the LR test of the hypothesis

$$H_0 : \gamma = 30.97 \text{ versus } H_1 : \gamma \neq 30.97. \tag{17}$$

(We have $N = 336$, because we avoid the measurements with zero waiting time). The Wilks statistics of the test of the hypothesis (17) has form

$$-2\ln\lambda = 2G_{336}\left(30.97 \sum_{i=1}^{336} W_i\right) - 2G_{336}(336),$$

where $W_i = T_i - T_{i-1} > 0$ are the waiting times (T_i are arrival times). We have $-2\ln\lambda = 2.94$ and exact p-value is 0.0865, hence the null hypothesis is not rejected for some reasonable level of significance. The Figure 1 displays the exact p-value[2] of the exact LR test for $30 < \gamma < 37$.

[2] The exact p-value is computed according to the formula $1 - F_{336}\left\{2G_{336}(\gamma \sum_{i=1}^{336} W_i) - 2G_{336}(336)\right\}$.

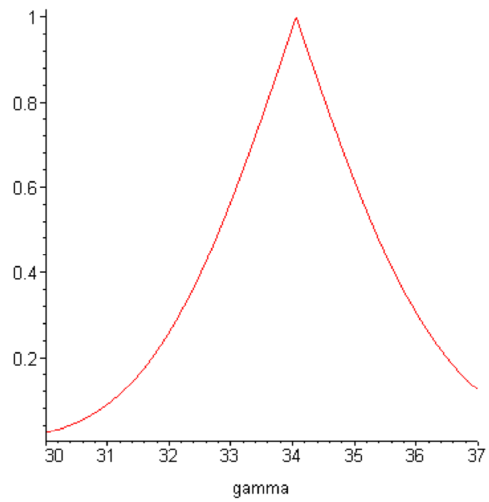

Fig. 1. The exact p-value.

The exact power $p(\gamma,\alpha)$ of the LR test of the hypothesis (17) based on the Wilks statistics of the exponential distribution at the significance level α at the point γ of the alternative has the form

$$p(\gamma,\alpha) = 1-F^{\Gamma}_{336}\left\{-\frac{336\gamma}{30.97}W_{-1}(-e^{-1-\frac{c_{\alpha,336}}{672}})\right\}+F^{\Gamma}_{336}\left\{-\frac{336\gamma}{30.97}W_{0}(-e^{-1-\frac{c_{\alpha,336}}{672}})\right\},$$

where $c_{\alpha,336}$ denotes the critical value of the test of the considered hypothesis on the level α (see [Stehlík 03, b]). The Figure 2 plots the power function for a range $30 < \gamma < 37$ and $c_{0.0865,336} = 2.94$.

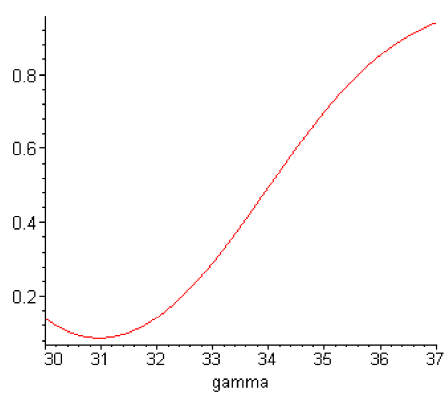

Fig. 2. The exact power.

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