Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), **1** (2005), No. 1

Network Analysis and Project Management – an Overview

FERDINAND GLIVIAK AND PETER SAKÁL

Abstract

The network analysis results have used for application tasks solution in project management. This paper gives an overview of basic network analysis problems and also some other ones in project management field. The article briefly describes the situation in finished software products for project management as well.

Mathematics Subject Classification 2000: *90B50, 90B10*
Additional Key Words and Phrases: network analysis, project management

INTRODUCTION

This paper is divided into 8 following parts: a project and its graphs, time analysis, resource analysis, cost analysis, traditional project management approaches, non – traditional project management approaches, current software tools and experiences with applications in Slovak republic.

1. A PROJECT AND ITS GRAPHS

A project – it is some complex or real task such as a house building, reconstruction of production hall, a product introduction, etc. The project consists of many partial activities. Some of them can be done by series and some of them by parallel. One activity is a set of labor hasing the same technical conditions to neighborhood. It is a practice to specify them in particular field. These activities are used in a project as one unit but they can be divided into the parts in the engineering practice.

We will study conjunctive – deterministic projects only, i. e. such project P that:

a) to every activity of P is given a positive real number as its duration,
b) given activity i of P can start only at that time if all immediate predecessors are done.

Table 1 describes the example of the project; see [J1, p. 77].

Table 1: The house building

Order	Task	Amount of time	Prec. tasks
1.	setting out the building estate	3	0
2.	delivering the building materials	2	0
3.	dining the foundations	2	1,2
4.	building the foundations	2	3
5.	building the walls	7	4
6.	building the roof	3	5
7.	covering the roof	1	6
8.	plumbing outside the house	3	4
9.	plasterworking outside	2	7,8
10.	putting the windows in	1	7,8
11.	putting the ceilings	3	5
12.	laing out the garden	4	9,10
13.	plumbing inside the house	5	11
14.	putting insulation on the walls	3	10,13
15.	painting the walls	3	14
16.	moving in	5	15

Other type of projects (e. g. conjunctive – stochastic) are mentioned in the literature but they have not such applications and therefore we will not talking about them.

We can allocate two different graphs into this project: (1) network graph N(P) of a project P, (2) activity graph A (P) of a project P.

In network graph N (P) we present every activity I of P by an oriented edge evaluated by its duration time d_i. More precise description of network graph construction is given in [SŠ1] and [GV]. We only notice that this graph N(P) contains this activity edges and also fictive edges that are useful for performance coordination between activities. Figure 1 give the network graph N (P) of the project P presented in Table 1.

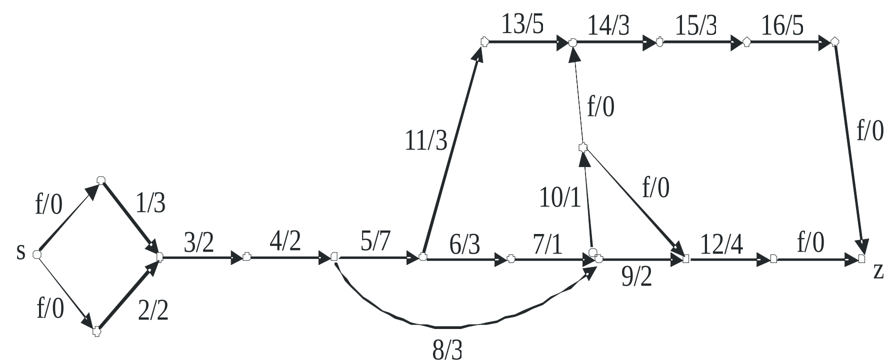

Fig. 1. Network graph N(P)

In activity graph A(P) we represent every activity i of P by a vertex evaluated by its time d_i . We join two activities i, j, by oriented edge [i, j], if the activity i is immediate predecessor of activity j. Moreover there are some fictive activities joining the start s the project P and the finish vertex z of P. Figure 2 give the activity graph A(P) of the project P from Table 1.

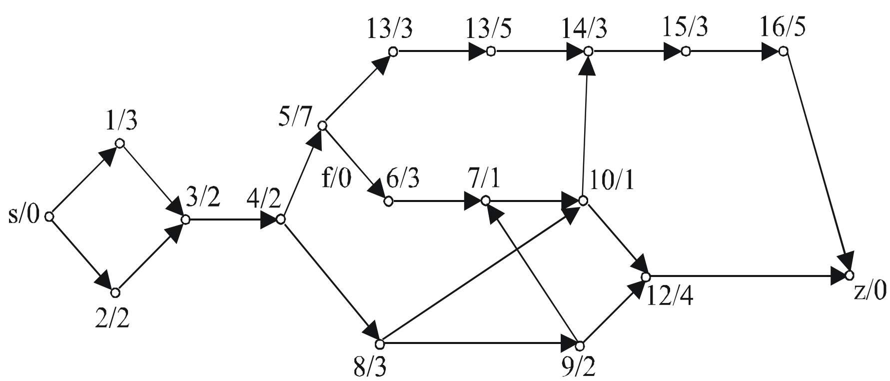

Fig. 2. Activity graph A (P)

Network graph N (P) arise in 1957 as first graph but now activity graph A (P) is the most used. Both these graphs must be cyclic and this fact is controlled automatically in current software system.

Time analysis, resource analysis and cost analysis are similar for both types of graphs. Therefore we have been talking only about activity graphs.

2. A TIME ANALYSIS

Every project starts at beginning (vertex s) and at time 0.Time in counted as discrete time form o during whole project duration.

In time analysis we use only time and topological relations between activities given by edges of graph A (P). We do not use resources. We suppose that there is enough resource of every needed type. For every activity i we compute the first possible beginning t_i of i and the last possible beginning T_i of i.

The first possible beginning t_i we compute by forward way (from start s to goal z) this way:

$t_s = 0$

$t_i = \max \{t_k + \text{duration activity } k / \text{ for all edges } [k,i] \text{ of } A (P)\}$

Specially, we receive at least possible end of the project t_z for fictive activity z (the goal of P). We put these time t_z equal to the last possible end of the project T_z. i. e. $t_z = T_z$.

We compute the last possible beginning T_i of activity i to the the step backward (from the goal z to the start s) in this way:

$T_z = t_z$

$T_i = \min\{T_j - \text{duration activity } i / \text{ for all edges } [i, j] \text{ of } A (P)\}$

Figure 3 shows numbers t_i and T_i for every activity i of the project given in Table 1.

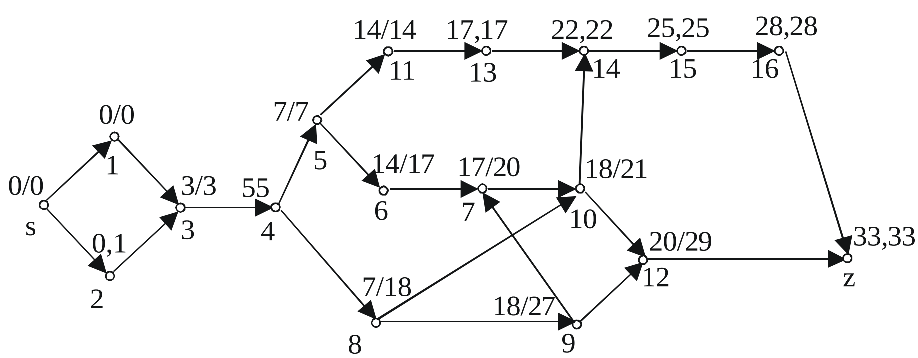

Fig. 3. The whole reserves and critical path

The difference $m_i = T_i - t_i$ between the latest possible and the first possible beginning of activity i is called the total reserve of activity i. In the current software system there are used these ones and also further reserves of activity i.

Critical path of Project P is the longest path staring at vertex s and finishing at vertex Z. The length of a path is the duration sum of activities lying on the path. Critical path of project P see at Figure 3. It is the path (s, 1, 3, 4, 5, 11, 13, 14, 15, 16, z). We mention that one project P can contain more than one critical path. Moreover, the sum of the whole reserve of any critical path of project P is 0.

Current software system for network analysis and project management illustrates the results of time analysis by Gantt diagram. It is a diagram with two axis x, y where on x is time and on axis y are different activities. For details see [U1], [T2]. Now we will talking about a resource analysis.

3. RESOURCE ANALYSIS

Generally, we need several resources for any project realization. Some of them depend on themselves. In the literature and also in the current practice it is supposed that resources are not mutually dependent. Then we often consider any resource separately what we will suppose in the future.

After time analysis we must count the demand of every resource in every discrete time. This is done for either left – hand side plan (i. e. all activities start in first possible time) or right – hand side plan (i. e. all activities start in latest possible time). The left – hand side plan is used more then right one. The result of this counting is step curve different form abscissa.

In resource analysis there are two professional problems [GV1], [SŠ1]:

a) to arrange the demand of every resource,
b) to assign constraint resource to activities.

In the first problem there are used three criteria of arranging of a resource [ZR1], [T2]:

1) The dispersion of the resource demand,
2) Maximal absolute deviation from average demand,
3) Maximal demand to a resource.

Let t = 1, 2, …, N is a discrete time in project P. Let Z (t) is a demand of resource Z in time t. Let $\overline{Z} = \frac{1}{N}\sum_{t=1}^{i} Z(t)$ is an average claim of resource Z per time unit. Then the demand dispersion of resource Z is $s_Z^2 = \frac{1}{N}\sum_{t=1}^{N}\left(Z(t)-\overline{Z}\right)^2$.

Criteria number one requires to minimize dispersion s_Z^2.

Criteria number two requires to minimize the relation $\max\limits_{t\in\langle 1,N\rangle}\left(Z(t)-\overline{Z}\right)$.

Criteria number three to minimize the relation $\max\limits_{t\in\langle 1,N\rangle}(Z(t))$.

There are different approaches this optimization task but none of them give an optimal solution of these tasks. They give approximation solutions only. For example this approximation algorithm of A. R. Burges for one resource is published in [K1]. The algorithm for one project for several independent resources providing one activity needs only one resource is published in [K1]. Levy, Thompson and Wiest study problem of several project and several independent resources, see [B2, p. 32].

Now we will study the second main problem. Let us suppose that arranging of every resource is done and we have less amount of some resource that is needed. Then the project realization is late. Algorithms allotting of delimit resource would allot the delimit resource at smallest possible delay of finishing project time.

Probably the best know algorithm for resource allotting is algorithm of J. E. Kelly solving this task for one project and one resource, [K1]. There is a generalization of this algorithm for one project, several independent resources and assumption that one activity needs only one resource. The problems of allotting resources for more projects would be discussed in part six (non – tradition approaches to project management). Now we will study one specific resource – money.

4. COST ANALYSIS

A project costs – there is money necessary for its realization. Cost analysis solves optimal synchronization between project costs and time of project realization. First of all we will talk about project cost, project duration and finally about basic facts of project shortening.

We will specify the activity costs and the project costs this way:

a) Direct activity costs usually contain the sum of the costs of materials equipments, salary of particular employees, etc. The direct costs of a project c_d are a sum of direct costs of all their activities.

b) Overhead costs on a project are usually given as overhead expenses per time unit c_o and their amount is given by multiplying of c_o by project duration.

c) Project extension deficits – they are determined concerning c_e (T – T_p), where c_e are project extension deficits per unit, T_p is a planning time of project duration an T is a real time of project duration.

d) Total costs c(T) for project realization per time T are given by:

$c(T) = c_d + c_o T + c_e (T - T_p)$

Duration of an activity usually is not a constant but it consists of upper duration t^u, lower duration t^e and further discrete values between them. Upper duration t^u of an activity is a activity duration with minimal resource requirements. We do not permit the extension of this duration behind the value t^u because of technological or other reasons.

Lowe duration t^e of an activity is a value that it is not possible reduce by further increasing of direct costs for the activity of using the same technology. If activity duration is the greatest $t_j = t^u$ than its direct costs are the lowest and equal to c^l. Vice – versa, if its duration is t^l than the direct costs are the greatest and equal to c^u. The relation between activity duration (axis x) and direct costs of their activity (axis y) is in the figure four. It is termed cost curve pre one activity. The cost curve of whole project had the similar shape, [U1], [SŠ1].

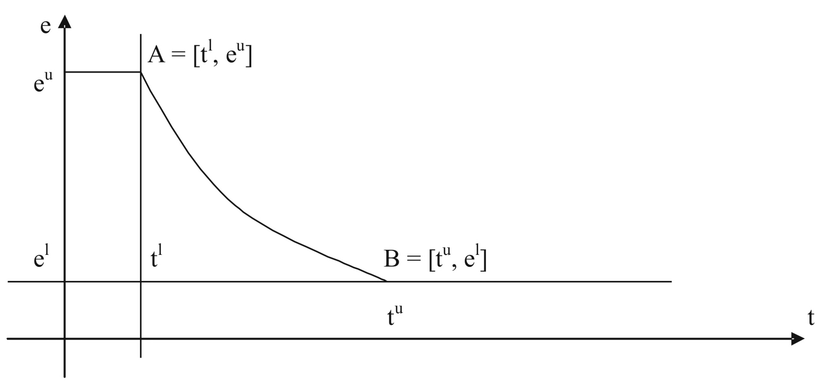

Fig. 4. Expense curve of an activity

In the literature it is usually suppose that cost curve of the project is continuous, nonlinear and convex. During the project extension the nonlinear part AB of project is a approximating by line passing the point A, B.

$c_i = -a_i t_i + b_i$, where $a_i = \dfrac{e^u - e^l}{t^u - t^l}$

Index a_i is constant of costs gradient and it expreses increase of costs per unit of extension of project duration- Well – known method of duration extension of the project is Weber's methods described in [U1]. Further methods of project abbreviation are described in [W1], [ZR1], and [B1]. All these methods give a heuristic solution of this main task: To determine the optimal duration of a project (ad hence duration of all activities) using this input data:

- upper and lower duration of every activity,
- upper and lower direct costs of every activity,
- planning time T_p of duration of this project,
- project extension deficits towards planning time T_p per time unit.

Current algorithms of solution for this main task are not published and moreover, they are business secret of particular firm.

5. TRADITIONAL APPROACHES TO PROJECT MANAGEMENT

Methods of network analysis are used in practice for project management. First of all we urge that a project is not only event but also its preparation, realization and documentation. Project management needs five different management activities [T2], [P1]:

1) defining of project goals,
2) planning of fulfilling project goals,
3) managing of project implementations,
4) monitoring of project activities process,
5) finishing of the project.

Now we notice only some main application problems concerning the previous five points. Detailed description of these activities would be longer and it is not a goal of this paper.

1) The definition of project goals would contain measurable material and verifying acceptation criteria to make sure the final output is really acceptable. Every goal have to contain exact and clear defined contents, scale (including the deviations) dead line, person who is responsible for goal fulfillment, way and periodicity of evaluation, see [P1], [B2].
2) Planning of goals fulfillment should have contain following properties, [T2]:
   a) identifying all activities necessary for project finishing,

b) it should have contains flow chart to set these activities,

c) defining necessary sources and time of their application,

d) it had the budget for every activity,

e) it contains back – up for unpredictable events,

f) the plan is credible for implementation and project managers, too.

The credibility of plan (property f) is important during the whole realization of the project. Reserves (property e) need some methodology for considering of project risks. These risks are uncovered before starting of the project and during the morning of project process. Some methodic are published and some of them are firm property only, [K2]. The main properties of these methodic are described in [T2]. The term – project risks relate with the term critical path described is the chapter network graphs, part 2.

3) A problem of project implementations managing and a problem of monitoring project activities process are detail propositions frame of project management. We will not describe them here but readers can receive some basic knowledge in [G1], [C1] and [SJ1].

4) The best reason for project finishing is fulfillment of all project goals defined at the project beginning. Necessity of all resources can change during the project realization but only by change process. Project success depends on many factors, e. g. clear and stable project goals, cognizant customer, experienced management, early project actualization, etc. (see e. g. [C1], [IB1]).

6. NON – TRADITIONAL APPROACHES TO PROJECT MANAGEMENT

A critical path of a project determines time necessary for project finishing. Every activity of a project can loose therefore there are insert some activity reserves into the project. They are based on pessimistic forecast of every activity duration. E. Godratt proposed to change this standard technique this way [G1].

<u>Time change:</u>

We will not estimate duration of every activity by pessimistic forecast but the most probable estimation. We will not assign the reserves to every activity but only:

- to the end of a critical path (it is called project buffer),
- to the end of every project branch concerning to a critical path (it is called joining buffer).

When the activity duration will be greater than their estimation, we will draw the rime from the nearest buffer. The time is not enough for multiproject management. Resources are usually narrow and are used in several projects at the same time. Then, very strong relations between projects as consequences off resources exist, there.

Resource change:

E. Goldratt suggested carrying – out everything that relate to one project resource, at once. More precisely, either bond of succession in activity graph or bond of using the same resource can give the dependence between two activities of one project. Then we have chains of sequentially depending activities. The longest chain from these chains is called critical chain. The critical chain is different term then the critical path. In critical chain, we work with time reserve by project buffer or joining buffer analogically as it was in the previous part when was concerning the time.

7. ACTUAL SOFTWARE POSSIBILITIES

In this part we give only necessary orientation facts and we will not promote any software system.

1. Software system MS Project, version 2000 of firm Microsoft has been using by all universities for a little tariff. This system is very useful for teaching of project management and for first professional experiments with a project.
2. Society for Project Management in Slovakia (SPPR) organizes a conference or seminar concerning new applications of project management, every year. Both authors of this paper take part in these conferences. There is possible to see the trends in application and also in using some software systems, see [I1].
3. Some software specialists are forecasting that there exist approximate 60 different software products for this area, now. We will not specify them only mention the names of that they are made good in our field. They are e. g. Super-project, Prochain, Primavera, Planisware. Promotional materials of all these systems give all their responsibilities and also their actual price in Slovak republic.

8. APPLICATION EXPERIENCES IN SLOVAC REPUBLIC

Results of network analysis have transformed to software systems for project management gradually. This process was an international one and now we mention several facts concerning realization in Slovakia.

1. In Slovakia, there are two firms – Slovakodata Bratislava and Aiten Trnava that have been dealing with project management more than 10 years. The both firms are approximately same large. Slovakodata Bratislava is solving about 15 parallel projects, see [K4]. Project management is one of the main activities of both firms, see [I4], [I5].

2. Besides them, there exist a lot of industrial applications of project. Many of them are presented on the conferences society SPPR. Authors and contents of these projects can be found in [I1]. The seat of Slovak Society for Project Management is in Trnava at Faculty of Materials Science and Technology.
3. Universities in Slovakia have very good background in this field. Specialists for projects management finish every year at Slovak University of Technology Bratislava (Faculty of Materials Science and Technology, Faculty of Engineering), Economical University Bratislava. There are similar study branch at other universities, e.g. Komenius University, Faculty of Mathematics, Physics and Informatics, Bratislava there is study branch mathematics – management.

We are mentioning some examples of various business projects solving at The Department of Industrial Engineering and Management of Faculty of Materials Science and Technology of Slovak University of Technique in Slovak republic:

- Time analysis of fuel changing in power plant V-1 Jaslovské Bohunice by CPM methodology for EC 1021 (1983),
- Time analysis of new product loading in to the production of National Coin Kremnica (1991),
- Time analysis of general revision of automatic mechanical batten machine ASP 400 (1991),
- Layout of universal model of EMS construction by ISO 14 000 in Chemosvit, a. s. Svit (1999),
- Application of project planning methods for construction of EMS in SE – EBO Jaslovké Bohunice (2000),
- Project planning utilization for EMS loading in TOPOS, a. s. Továrniky (2004).

You can find the other examples in actual information of SPPR, see [I1].

ACKNOWLEDGMENTS

The authors would like to thank to Dipl. Ing.Helena Vidová – senior lecture of Department of Industrial Engineering and Management, Faculty of Materials Science and Technology Trnava, Slovak University of Technology for technical and lingual correction.

REFERENCES

[B1] Blecha V.: Síťová analýza ve stavebníctví. Dúm techniky ČSVTS, Praha, 1989, (in Czech).

[B2] Bullová K.: Sieťová analýza a riadenie projektov. Diploma work for Mgr., supervisor F. Gliviak. Comenius University, FMFI, Bratislava, 2004, (in Slovak).

[C1] Čambál M. – Holková A. – Horňák F.: Manažérstvo podniku. Slovak University of Technology, MtF, Trnava, 2000, (in Slovak).

[G1] Goldratt E. M.: Kritický reťazec, Inter Quality, Praha, 1999, (in Czech, translation from English).

[G2] Gros I.: Kvalitatívne metódy v manažérskom rozhodovaní. Grada, Praha, 2003, (in Czech).

[GV1] Gliviak F. – Vadkerti P.: Sieťová analýza a manžment projektov. Univerity of St. Cyrili and Methodi, FPV, Trnava, 2001, (in Slovak).

[GY1] Gross J. – Yellen J.: Graph Theory and its Applications. CRC Press, New York, 1998.

[IB1] Ivaničová Z. – Brezina I. - Pekár J.: Operačný výskum. Economical university, Bratislava, 2002, (in Slovak).

[J1] Jungnickel D.: Graphs, Networks and Algoritms. Springer, Berlin, 1999.

[K1] Klusoň V.: Kritická cesta a PERT v řídíci praxi. SNTL, Praha, 1968, (in Czech).

[K2] Kubiš J: Existenčné riziká projektu a k nemu viazaných subjektov. In: Tvorba SW 2002 (celoštátna konferencia), Ostrava, Česká společnost pro systémovou integraci 2002, 109 – 116, (in Slovak).

[K3] Kubiš J.: Cykly v projektoch. In: tvorba SW 2003(celoštátna konferencia), Ostrava, Česká společnost pro sysémovou integraci, 2003, 63 – 70, (in Slovak).

[K4] Kubiš J.: Analýza výkonov SW firmy. In: Tvorba SW 2004, Ostrava, Česká společnost pro sysémovou integraci, 2004, 263 – 270, (in Slovak).

[P1] Projektový manažment a jeho využitie v praxi. (Metódy a nástroje). Proc. of the seminar, Slovak University of Technology, MtF, Trnava, 2004, (in Slovak).

[SŠ1] Sakál P. –Štrpka A.: Operačná a systémová analýza. Slovenská technická univerzita, Strojnícka fakulta, Bratislava, 1982, (in Slovak).

[SJ1] SakálP. – Jerz V.: Operačná analýza v praxi manažéra. SP Synergia, Trnava, 2003, (in Slovak).

[T1] Trávnik I. – Vlach J: Sieťová analýza. ALFA, Bratislava, 1974, (in Slovak).

[T2] Trávnik I.: Projektmanažment. Slovenská techniká univerzita, Stavebná fakulta, Bratislava, 2001, (in Slovak).

[U1] Unčovský L. a kol.: Modely sieťovej analýzy. ALFA, Bratislava, 1991, (in Slovak).

[ZR1] Zuchovicky S. J.- Radčikova J. A.: Matematické matódy síťové analýzy, SNTL, Praha, 1972, (in Czech, taranslation form Russian).

[W1] Walter J.- Vejmola S.- Fiala P.: Aplikace metód síťové analýzy v řízení a plánovaní. SNTP, Praha, 1989, (in Czech).

Internetové zdroje:

[I1] http://www.sppr.sk

[I2] http://www.goldratt.cz

[I3] http://www.goldratt.co.uk

[I4] http://www.slovakodata.sk

[I5] http://www.aiten.sk

Network Analysis and Project Management – an Overview

F. Gliviak,
Faculty of Mathematics, Physics and Informatics, Comenius University Bratislava,
Mlynská dolina, 842 48 Bratislava, Slovak Republic
e-mail: gliviak@fmph.uniba.sk

P. Sakál,
Faculty of Material Science and Technology, Slovak University of Technology,
Paulínska 16, 917 04 Trnava, Slovak Republic
e-mail: sakal@mtf.stuba.sk

Received October 2004