Journal of Applied Mathematics, Statistics and Informatics (JAMSI), **1** (2005), No. 2

Exact Decision Methods in Project Management *

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Abstract

This paper is figures out possibility of use other quantitative methods by decision making in project management, namely calculus of probability and optimalization models.

Mathematics Subject Classification 2000: 90B50
Additional Key Words and Phrases: exact decision methods, project management

1. INTRODUCTION

Among basic issues in project management belongs also determination about size of projected production capacities and decision which of prepared projects should be realised so that we could achieve maximal effect of realization with minimum risk. We can achieve solution of these present decision problems in project management only with difficulties without use of exact decision methods, which are based on mathematical principles and relations. Exact approach in decision and IT development has been a determination impulse and a base for development of quantitative approaches also in project management. Most important use of quantitative methods ensemble in project management have methods of network analysis, especially in area of network planning.

In this article are analysed and modelled three various situations with use of three different approaches.

By reason of objectivisation is in this paper proposed for selection method which is combination of mean value/ average value method and methods of variation. The suggested method enables to choose variant of business project that is optimal from annual profit expectation aspect together with minimal risk. For solving of capacity optimalization of projected production units it is used a linear optimalization model. For optimalization of project portfolio is suggested a optimalization model of bivalent goal programming that enables on the ground of estimated goal (to achieve required value of year profit) to optimalise not only project portfolio but also disposable resources by

* This paper was elaborate by solution of the project VEGA MŠ 1/2578/05 "The Analysis of Actual Trends of Project Management in the World, the Research of Recent Situation in the Slovak Republic and the Design its Exploitation in the Conditions of Slovak Republic."

variables y_i^+ (i = 1, 2, …, m), that presented values how we should optimal enhance amounts of disposable resources so that estimated goal could be achieved.

2. DETERMINATION OF OPTIMAL PROJECT VARIANT

We suppose that enterprise has its business project which goal is to build up production unit for production of new product in several variants and should decide which variant will be realised. Economic efficiency of variants can be considered upon various criteria. The enterprise decided to choose as a criteria annual profit of product sale.

Height of achievable annual profit affect n factors which we name with symbols F_1, F_2, ..., F_n.

These factors can have several levels. First factor can have one, two to r levels which we name with symbols $U_{11}, U_{12}, ..., U_{1r}$.

Second factor can have one, two to s levels which we name with symbols $U_{21}, U_{22}, ...,U_{2s},$ etc. n factor can have one, two to t levels, which we name with symbols $U_{n1} ,U_{n2} , ... , U_{nt}$.

Enterprise has prepared m variants of business project which we name with symbols

$$V_1 ,V_2 , ... ,V_m ,$$

of which it should choose one so that it will achieve maximal annual profit from product sale provided that by lower demand than is a production capacity, it will decrease production to demand level and so it will not produce for storage.

We suppose that business experts estimated probabilities of several factors levels appearance, which we name with symbols

$$p_{11} , p_{12} , \cdots ,p_{1r}$$

for first factor levels,

$$p_{21} , p_{22} , \cdots , p_{2s}$$

for second factor levels etc. as many as

$$p_{n1} , p_{n2} , \cdots ,p_{nt}$$

for n-th factor levels.

Goal of enterprise is to choose such variant of business project that will help to maximal annual profit with minimal risk.

$$E(z_i) = z_{i1} P_1 + z_{i2} P_2 + z_{i3} P_3 + ... + z_{i, (r.s. ... . t)} P_{(r.s. ... . t)} , \quad (i = 1, 2, ... , m) \qquad (1)$$

where z_{ij} is a expected annual profit that will be achieved if the enterprise will choose project variant V_i and it is combination of factor levels j with probability P_j, we calculate a total number of factor levels combination as a product of factor levels, i.e. ($r.s. ... .t$).

If we declare combination of factor levels as $A_1, A_2, \ldots, A_{(r.s. \ldots . t)}$, than we can declare compound appearance:

$$\begin{aligned} A_1 &= U_{11} \cap U_{21} \cap \ldots \cap U_{n1} \\ A_2 &= U_{11} \cap U_{22} \cap \ldots \cap U_{n1} \\ &\ldots\ldots\ldots\ldots \\ A_{(r.s. \ldots . t)} &= U_{1r} \cap U_{2s} \cap \ldots \cap U_{nt} \end{aligned} \tag{2}$$

And their occurrence probabilities $P_1, P_2, \ldots, P_{(r.s. \ldots . t)}$:

$$\begin{aligned} P_1 &= p_{11} p_{21} \ldots p_{n1} \\ P_2 &= p_{11} p_{22} \ldots p_{n1} \\ &\ldots\ldots\ldots\ldots \\ P_{(r \times s \times \ldots \times t)} &= p_{1r} p_{2s} \ldots p_{nt} \end{aligned} \tag{3}$$

Then we should find out supposed values of profit variation:

$$D(z_i) = [z_{i1} - E(z_i)]^2 P_1 + [z_{i2} - E(z_i)]^2 P_2 + \ldots + [+ z_{i,(r.s. \ldots . t)} - E(z_i)]^2 P_{(r.s. \ldots . t)} \tag{4}$$

$(i = 1, 2, \ldots, m)$

To calculate the optimal project variant we arrange necessary information in a table. In the table we can find values of expected annual profit of each project variant for several factor levels combination, mean values and variations of annual profit for each project variant.

Project Variants	Combination of Factor Levels and their Probabilities				Mean value of Annual Profit	Values of Annual Profit Variation
	A_1	A_2	. . .	$A_{(r.s. \ldots .t)}$		
	P_1	P_2	. . .	$P_{(r.s. \ldots .t)}$		
V_1	z_{11}	z_{12}	. . .	$z_{1,(r.s. \ldots .t)}$	$E(z_1)$	$D(z_1)$
V_2	z_{21}	z_{22}	. . .	$z_{2,(r.s. \ldots .t)}$	$E(z_2)$	$D(z_2)$
. . .	. . .	. . .	. . .	. . .	. . .	. . .
V_m	z_{m1}	z_{m2}	. . .	$z_{m,(r.s. \ldots .t)}$	$E(z_m)$	$D(z_m)$

Based on values in last and last but one column of table we can calculate an optimal project variant by mean value and variation method. By means of this method is optimal the variant which has biggest mean value (average value) and the smallest risk variation of chosen criteria (i.e. expected annual profit) in comparison with other variants or the variant which has better one of these characteristics (mean value or variation) than all other variants and its second characteristic is not worse than for other variants.

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Topics include, but are not limited to: complexity of algorithms, AI, computer graphics, simulations, stochastic methods, database theory, graph theory, to name a few.

3. DETERMINATION OF OPTIMAL PRODUCTION CAPACITIES OF PROJECTED PRODUCTION UNITS

We suppose that the enterprise would like to extend its production program by new product. Production units for production of this product can be build on p various places. For this production there are needed m various raw materials. Unit consumption of t-th raw material we name s_t (t=1, 2, ..., m). We can obtain every raw material from several sources. There exist n_1 sources of first raw material with capacity S_{1i} (i = 1, 2, ... , n_1), n_2 sources of second raw material with capacity S_{2j} (j = 1, 2, ... , n_2), etc. thus n_m sources of m-th raw material. Products are supplied to customers, their annual requirements are P_1, P_2, ..., P_s. Firm management should decide, what kind of production capacities should be projected for production units on several places, so that annual costs for raw materials and product transport and annual production costs were minimal.

Now we can formulate this issue in mathematical way as a linear optimalization model, that consists of objective function and boundary conditions. Value of objective function will be sum of annual costs for raw material and products transport. We assume that transport costs are proportional to transported raw material or products and costs for production unit are constant.

If we name as symbol

- c_{tkr} costs for transport of t-th raw material from k-th source on r-th place,
- x_{tkr} transported amount of t-th raw material from k-th source on r-th place,
- d_{rg} costs for production unit from r-th production place to g-th customer,
- y_{rg} amount of transported production units from r-th production place to g-th customer,
- c_r costs of production unit,
- Y_r annual production volume on r-th place,

Then we can write objective function:

$$z = \Sigma\, c_{1ir}x_{1ir} + \Sigma\, c_{2jr}x_{2jr} + ... + \Sigma\, c_{mvr}x_{mvr} + \Sigma\, d_{rg}y_{rg} + \Sigma\, c_r\, Y_r \qquad (5)$$

where first m sums are annual transport costs for several raw materials, last but one sum is annual costs for production transport and last sum is annual production costs.

The goal is to find out such nonnegative values of variables x_{tkr}, , y_{rg} a Y_r , when the objective function will have minimum and boundary conditions fulfilled:

$$\sum_r x_{tkr} \le S_{tk}, \qquad (t = 1, 2, ..., m\ ;\ k = 1, 2, ..., n_t) \qquad (6)$$

i.e. from each raw material source we can transport at most so much raw material as it is its capacity. Boundary conditions (6) are $n_1 + n_2 + ... + n_m$.

$$\sum_k x_{tkr} = s_t\, Y_r\ , \qquad (t = 1, 2, ..., m\ ;\ r = 1, 2, ..., p) \qquad (7)$$

i.e. we suppose that each production place will have so much raw materials as it needs for its production. Number of these boundary conditions is m x p.

$$\sum_g y_{rg} = Y_r\ , \qquad (r = 1, 2, ..., p) \qquad (8)$$

i.e. we suppose that each production place will not produce for storage.

$$\sum_r y_{rg} = P_g, \qquad (g = 1, 2, ..., s) \qquad (9)$$

i.e. customers requirements have to be fulfilled.

$$q_r \le Y_r \le Q_r\ , \qquad (r = 1, 2, ..., p) \qquad (10)$$

i.e. on r-th place can be projected production capacity maximal Q_r units and minimal q_r, $0 \le q_r \le Q_r$.

Relations (5) to (10) build together the linear optimalization model. Optimal solution of this model are such values of variables x_{tkr} , y_{rg} a Y_r , that will minimize value of function (5) and fulfil conditions (6) - (10). Values of variables Y_r are annual production capacities of projected production units.

4. OPTIMALIZATION OF PROJECT PORTFOLIO

Optimalization of project portfolio means to choose such set from multitude of prepared projects which realisation is with acceptation of boundary conditions possible and the most advantageous. This is the optimal project portfolio.

We suppose that issue of organisation is to choose for realisation such project set from multitude Of a prepared projects their realisation secures that annual profit of realisation will be as near as possible to expected annual profit z_p, and it could be realised by m boundary source conditions.

It is not goal to choose such set of projects, that will mean maximal possible annual profit but the one which will mean observance of determined annual profit. Thus we can use for solution of this issue following model of goal programming:

$$z^- + z^+ = \min \tag{11}$$

conditions:

$$z_1x_1 + z_2x_2 + ... + z_nx_n + z^- - z^+ = z_p \tag{12}$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n &\leq b_2 \\ &\ldots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n &\leq b_m \end{aligned} \tag{13}$$

$x_j =$ 1 or 0 for $j = 1, 2, ..., n$; $z^-, z^+ \geq 0$

Where: z^- is variable, it is sum which is missing to value z_p,

z^+ is variable, it means sum about which is higher than z_p,

z_j is annual profit of realisation of j-th project,

a_{ij} is consumption of i-th source for realisation of j-th project,

b_i is disposal amount of i-th source,

x_j is binary variable, if $x_j = 1$ in optimal solution 1, j-th project is in optimal set, if $x_j = 0$, j-th project does not belong to optimal set.

Variables z^- and z^+ are contradictory conditions, in optimal solution is at least one of them 0.

If it is not allowed not to get required value of z_p, so we should to find out such project set that will secure required annual profit, then this issue could be inconsistent, when disposable amount of at least one source is not enough for project realisation. It can help, when we write boundary conditions in this way (12). For solution of this issue is to use following model of goal programming:

$$z^+ + y_1^- + y_1^+ + y_2^- + y_2^+ ... + y_m^- + y_m = \min \tag{14}$$

conditions:

$$z_1x_1 + z_2x_2 + ... + z_nx_n - z^+ = z_p \tag{15}$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + y_1^- - y_1^+ &= b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + y_2^- - y_2^+ &= b_2 \\ &\ldots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n + y_m^- - y_m^+ &= b_m \end{aligned} \tag{16}$$

$$x_j = 1 \text{ or } 0, \text{ for } j = 1, 2, ..., n \text{ ; } z^+ \geq 0 \text{ ; } y_i^-, y_i^+ \geq 0, \text{ for } i = 1, 2, ..., m \tag{17}$$

Where optimal variables values y_i^- (i = 1, 2, ..., m) are reserves in several sources and optimal variables values y_i^+ (i = 1, 2, ..., m) are value how we should increase disposable amounts of sources, to enable realisation projects set, which will mean achievement of required annual profit z_p . At least one of variables y_i^-, y_i^+ (i = 1, 2, ..., m) is in optimal solution 0.

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Received October 2005