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On two operations over intuitionistic fuzzy sets

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Abstract

Two new operations with IF sets are defined. They are in some sense inverse to the sum and the product.

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Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the empty IFS and the unit IFS by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Different relations and operations are introduced over the IFSs. Some of them are the following

$$\begin{array}{lcl}
A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A + B & = & \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\
A.B & = & \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\
A@B & = & \{\langle x, (\frac{\mu_A(x) + \mu_B(x))}{2}, \frac{(\nu_A(x) + \nu_B(x))}{2}\rangle | x \in E\}.
\end{array}$$

In this short remark we introduce two new operations, defined over IFSs. They are analogous of operations “subtractionänd “division".

DEFINITION 1. *Denote by* $\alpha(A,B)$ *the set of all* $x \in E$ *such that*

$$\mu_B(x) \le \mu_A(x), \nu_A(x) \le \nu_B(x), \mu_B(x) < 1, \nu_B(x) > 0,$$

$$\nu_A(x)(1-\mu_B(x)) \le \nu_B(x)(1-\mu_A(x))$$

Then

$$A - B = \{\langle x, \mu_{A-B}(x), \nu_{A-B}(x)\rangle | x \in E\},$$

where

$$\mu_{A-B}(x) = \begin{cases} \dfrac{\mu_A(x)-\mu_B(x)}{1-\mu_B(x)}, & \text{if } x \in \alpha(A,B) \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{A-B}(x) = \begin{cases} \dfrac{\nu_A(x)}{\nu_B(x)}, & \text{if } x \in \alpha(A,B) \\ 1, & \text{otherwise} \end{cases};$$

The motivation is the following. Consider $C = (\mu_C, \nu_C)$ such that $B + C = A$. Then

$$\mu_B + \mu_C - \mu_B\mu_C = \mu_A, \nu_B\nu_C = \nu_A.$$

Of course, there are some problems with finding such C in general.

LEMMA 1. $\mu_{A-B}(x) + \nu_{A-B}(x) \le 1$ *for any* $x \in E$.

PROOF. If x is not in $\alpha(A,B)$, then the left side is 0 and the inequality holds. If $x \in \alpha(A,B)$, then

$$\mu_{A-B}(x) + \nu_{A-B}(x) = \frac{\mu_A(x)+\mu_B(x)}{1-\mu_B(x)} + \frac{\nu_A(x)}{\nu_B(x)} =$$

$$= \frac{(\mu_A(x)-\mu_B(x))\nu_B(x) + \nu_A(x)(1-\mu_B(x))}{(1-\mu_B(x))\nu_B(x)} \le$$

$$\le \frac{(\mu_A(x)-\mu_B(x))\nu_B(x) + \nu_B(x)(1-\mu_A(x))}{(1-\mu_B(x))\nu_B(x)} = 1.$$

□

THEOREM 1. *For every two IFSs* A *and* B:
(a) $A - A = O^*$,
(b) $A - O^* = A$.

For to prove $(A-B)+B = A$ we need some special assumptions on A and B.

DEFINITION 2. *We shall say that an IFS B is regular, if $\mu_B(x) < 1$ and $\nu_B(x) > 0$ for any $x \in E$ If $A, B$are two IFSs, then*

$$A \prec B$$

if and only if

$$\mu_B(x) \leq \mu_A(x), \nu_A(x) \leq \nu_B(x), \nu_A(x)(1 - \mu_B(x)) \leq \nu_B(x)(1 - \mu_A(x))$$

for any $x \in E$.

The preceding relation is motivated by the inequality

$$\frac{\nu_A(x)}{\nu(B(x)} \leq \frac{1 - \mu_A(x)}{1 - \mu_B(x)}.$$

The relation $A \prec B$ holds, whenever $A \subset B$ and A, B are induced by fuzzy sets, i.e. $\nu_A = 1 - \mu_A, \nu_B = 1 - \mu_B$.

THEOREM 2. *If B is regular, and $B \prec A$, then $A = (A - B) + B$.*

PROOF. In this case $\alpha(A, B) = E$. Therefore

$$\mu_{A-B} + \mu_B - \mu_{A-B}\mu_B = \frac{\mu_A - \mu_B}{1 - \mu_B}(1 - \mu_B) + \mu_B = \mu_A,$$

$$\nu_{A-B}\nu_B = \frac{\nu_A}{\nu_B}\nu_B = \nu_A.$$

□

DEFINITION 3. *For any IFSs A, B define $A : B = \overline{\overline{A} - \overline{B}}$.*

THEOREM 3. *For every IFSs A and B*

$$(a) A : A = E^*,$$

$$(b) A : E^* = A.$$

If $\overline{B}$ is regular and $\overline{B} \prec \overline{A}$, then

$$(c)(A : B).B = A.$$

PROOF. Evidently $A : A = \overline{\overline{A} - \overline{A}} = \overline{O^*} = E^*, A : E^* = \overline{\overline{A} - \overline{E^*}} = \overline{\overline{A} - O^*} = \overline{\overline{A}} = A$. Finally the regularity of $\overline{B}$ and the relation $\overline{B} \prec \overline{A}$ imply

$$(A : B).B = \overline{\overline{A} - \overline{B}}.\overline{\overline{B}} = \overline{(\overline{A} - \overline{B}) + \overline{B}} = \overline{\overline{A}} = A.$$

□

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