Set Theory Based Analysis of Atonal Music

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Abstract
The article presents basic posssibilities of interdisciplinary oriented research in analysis of atonal music through set theory applications. The main question is, if this application is usefull in finding new musicological relevant knowledge.

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Additional Key Words and Phrases: coding and segmentation of music, pc sets, melodic and harmonic patterns, Z-sets, pc set complex, similarity of patterns

1. INTRODUCTION

1.1. Aims and Scope

Formalistic type od analysis is oriented on translation or coding music to symbols or patterns. This attempt is motivated by the belief, that the music form as a macro-structure is based on the same type of organization as its micro-structures. That means, that the formal structure of musical composition is possible to interpret by the same rules on more hierarchical levels.

Similar motivation from thematic or harmonic point of view is possible to find in Rudolf Reti´s [1] or Heinrich Schenker´s [2] theories.

Analysing of atonal, dodekafonic and serial music is not possible from the melodic or harmonic point of view in a traditional sense. In this kind of music there are no classical motives and themes or triads and other seventh chords, there are no harmonic functions and/or change of tension and motion, based on consonant-disonanct contrast.

For these reasons a new theory of classification of structures (both horizontal and vertical) was developed to use in analysis of atonal music. It´s main author, Allen Forte [3], started to segment musical pieces into new type of segments, named „ ***pitch class sets“.***

Nicolas Cook [4] evaluated this way of analysis and interpretation of music as a static one, because of it does not take care on perception and experience of music.

This method differs from Schenker´s one in a very basic question, so also differs in a main goal. Schenker´s method tends to answer a question, „*how is music experienced as directed motion?“,* while Forte´s method asks another question, „*how should be the music recoded to make it´s unity evident?“* p.122 [4].

Forte and other formalists tried to find a new, precise and objective method for analysis of score, which excludes the pure subjective speach about our experience of music. They thought this method should be not only objective, but also a scientific one.

Forte has an idea, that in atonal music there are basic types of pitch-structures, which are similar and the whole composition is created from some basic pitch-structures or formations. These formations are possible to compare with chords in tonal music (triads and seventh chords) and could establish the framework for description, interpretation and explanation in analysis of atonal composition.

1.2. Pitch class set

Forte has introduced a new term *pitch class set,* shortened as **pc set.** It is a set of numbers, which represent pitches in a variety of tones of well tempered scale, i.e. there are 12 various pitches, and a set of pitches in amount of 3 to 9 is basic. If the lowest pitch of this basic set is 0, every higher pitch is represented with a number of halftones, which are the differences between them. The occurance of the elements in one set is ascending from the lowest to the highest pitch, and it does not depend on the real occurance of them in the musical segment. Even if the lowest pitch would be the last one in the segment, and the melody of it would be descending, the pc set would be ordered from the lowest pitch to the highest one (but in the same octave and in the smallest possible difference).

The method tends to find the smallest difference also in inversions modulo 12. In tonal music it means, that every inversion and position of triad should be converted to the same pc set.

On the end of his book *The Structure of Atonal Music* [3] Forte created a library of pc sets. Every set there is identified with two numbers: 1. – an amount of elements (3 – 9) and 2. an ordinal number pc set library. The identification of the pc set in the Figure 1 is 3-3 [014], so there are 3 elements, the third pc set in Forte´s library, basic difference between the first lowest and the second lowest (modulo 12) element is one halftone, between the second and the third (lowest) element is 4 halftones.

Another formalist, Robert D. Morris [5] has found out, that one pc set built out from 3 pitches, which are represented with pc set [014], could be written in score with many variants (inversions), as we can se in the Figure 1.

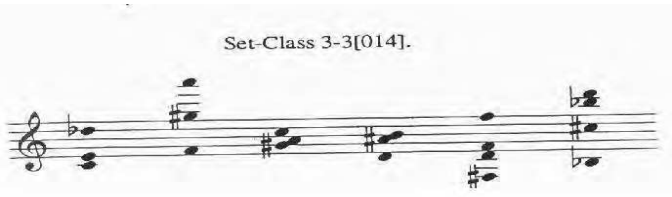

Figure 1: the variability of one pc set, Figure cited from [6], p. 213

If we analyse any composition, the basic and most problematic is the decision, where is the right point, where to segment the composition. As we can see in the Figure 2, also as short piece as of 2 measures shows two possibilities of segmentation. We can choose a set of 4 elements - eight in the library, or a set of 6 elements - 76th in the library.

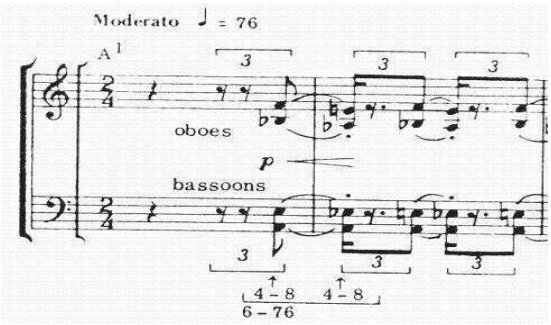

Figure 2: PC sets in a part of Stravinskij: Excentrique. Figure cited from [4], p. 141, measure 1-2.

As Cook pointed out, the only really objective and rigorously manner to achieve the full and not subjective segmentation is to segment in every possible way, so the pure and short melody of 7 pitches should be segmented to many various pc sets, as we can see in the Figure 3.

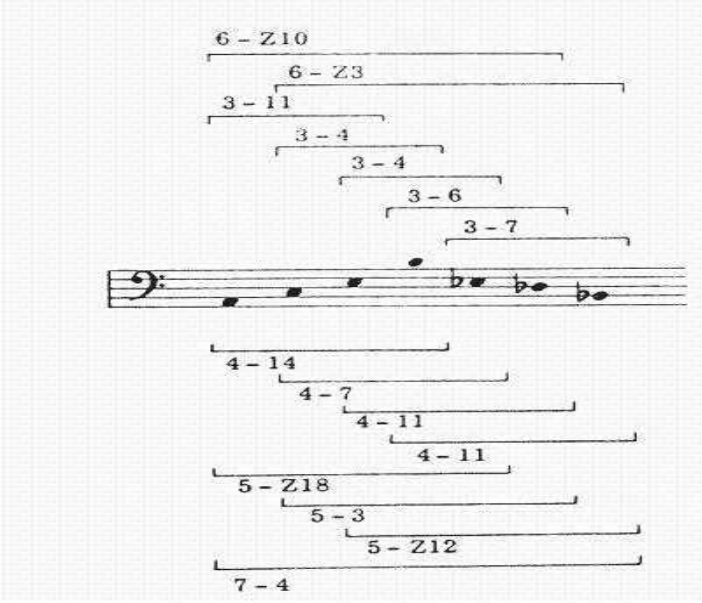

Figure 3: Imbrication – full segmentation of one part of melody., cited from [4], p. 147

This way of analysis into pc sets brings so many (hundreds) pc sets from one composition, that we can dispute the topic and the significance of this result for other musicological knowledge and research.

The Forte´s theory includes other enumerations. One points out an intervalic potention of a pc set. This is represented with a six elements-vector, in which every digit (numeral) enumerates the number of possible intervals of halftones (ordered from smallest of 1 halftone, to biggest of 6 halftones, greater are possible to represent with it´s inversion, i.e. 7 is the same in inversion, as 5, 8 is inversion of 4, etc.).

1.3. Z-sets, pc set complex

The vektor 012111 is for instance intervalic potention of classical structure of dominant seventh chord. It´s pitches are $g - h - d - f$, so 0 minor seconds, 1 possible major second, 2 possible minor thirds, one major third and one diminished fourth are possible to find in this structure). Pc sets, which have the same intervalic vector, create the new set of **Z-sets.**

Forte is working - except basic pc sets and Z-sets - also with complement sets and pc set complexes. In **pc set complex** are connected those pc sets, which are parts of one superset.

The full amount of basic pc sets in Forte´s library is 208, the amount of pc set complexes is only 114.

If the whole composition is segmented and sorted to Z sets and pc set complexes, then it is possible to start with other set operations.

1.4. Conclusion

The user of this analytical method has to work with respect to the reality of music, The results of these operations should have any sense not only from mathematical point of view, but they should help find out either any musicological relevant new information or other result (musical feature, property, syntactical rule, etc.). The similarity with classical tonal structures (as triads and other chords) is problematic, because the experience of perception of classical chords is basen on well-audible consonant or disonant character of them. The numeral character of pc sets doesn´t differ in sense of reception, as this kind of analysis is not clearly correlated with an experience of audience.

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