ON THE SOLUTION OF GENERALIZED FREDHOLM TYPE INTEGRAL EQUATIONS

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Abstract

In the present paper, we first solve an integral equation whose kernel involves the product of a general class of multivariable polynomials and the generalized Mellin Barnes type contour integral (popularly known as the $\overline{H}$ -function) by the application of Mellin transform theory. On account of the general nature of kernel of our main integral equation, the solution of a number of integral equations can be obtained as its special cases. For the sake of illustration, we obtain here the solutions of three simpler integral equations involving the kernels- $\phi(z,p,\eta)$, $S_V^U(x)$, $L_V^{\alpha}(x)$ and $\overline{J}_{\lambda'}^{\nu',\mu'}(x)$ which are also new and of interest.

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1. INTRODUCTION

A large number of Fredholm type integral equations involving various polynomials or special functions as their kernels have been studied by several researchers. An extensive and systematic presentation has been given in the book by Srivastava and Bushman [11].

The main object of this paper is to derive an exact solution of the following Fredholm type integral equation

$$\int_0^{\infty} y^{-1}\, u\left(\frac{x}{y}\right) f(y)\,dy = g(x) \qquad (x>0) \tag{1}$$

where g is a prescribed function , f is an unknown function to be determined and the kernel u(x) is given by

$$u(x) = S_V^{U_1,\ldots,U_k}[z_1 x^{\rho_1},\ldots,z_k x^{\rho_k}]\, \overline{H}_{P,Q}^{M,N}\left[tx^{-\lambda} \,\middle|\, \begin{matrix} (e_j,E_j;\epsilon_j)_{1,N},\,(e_j,E_j)_{N+1,P} \\ (f_j,F_j)_{1,M},\,(f_j,F_j;\Im_j)_{M+1,Q} \end{matrix} \right] \tag{2}$$

where $S_V^{U_1,\ldots,U_k}$, the general class of multivariable polynomials, is as given by Srivastava and Garg [12, p.686, Eq.(1.4)] and defined as

$$S_V^{U_1,\dots,U_k}[x_1,\dots,x_k] = \sum_{\substack{R_1,\dots,R_k=0 \\ \sum_{i=1}^{k} U_i R_i \le V}} (-V)_{\sum_{i=1}^{k} U_i R_i} A(V;R_1,\dots,R_k) \frac{x_1^{R_1}}{R_1!} \cdots \frac{x_k^{R_k}}{R_k!} \tag{3}$$

where $U_1,\dots,U_k$ are arbitrary positive integers, $V = 0,1,2,\dots$ and the coefficients $A(V, R_1,\dots,R_k)$ are arbitrary constants, real or complex. Further, $\overline{H}$ -function, a generalization of Fox H-function introduced by Inayat Hussain [7] and studied by Buschman and Srivastava [2] and others, is defined and represented in the following manner :

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[z \left| \begin{matrix} (e_j,E_j;\epsilon_j)_{1,N},(e_j,E_j)_{N+1,P} \\ (f_j,F_j)_{1,M},(f_j,F_j;\Im_j)_{M+1,Q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi\omega} \int_L \overline{\phi}(\xi) z^{\xi} \, d\xi \tag{4}$$

where
$$\overline{\phi}(\xi) = \frac{\prod_{j=1}^{M} \Gamma(f_j - F_j\xi) \prod_{j=1}^{N} \{\Gamma(1-e_j+E_j\xi)\}^{\epsilon_j}}{\prod_{j=M+1}^{Q} \{\Gamma(1-f_j+F_j\xi)\}^{\Im_j} \prod_{j=N+1}^{P} \Gamma(e_j - E_j\xi)} \tag{5}$$

and the contour L is the line from $c-\omega\infty$ to $c+\omega\infty$, suitably indented to keep the poles of $\Gamma(f_j - F_j\xi)$, $j = 1,2,\dots,M$ to the right of the path and the singularities of $\{\Gamma(1-e_j+E_j\xi)\}^{\epsilon_j}$, $j = 1,2,\dots N$ to the left of the path.

The other details about $\overline{H}$ -function can be seen in the paper cited earlier. Evidently, if we take ϵ_j $(j=1,\dots,N)$ and $\Im_j (j = M+1,\dots,Q)$ equal to unity, the $\overline{H}$ -function reduces to well-known Fox H-function [8].

The following sufficient conditions for the absolute convergence of the integral defined in equation (4) have been recently given by Gupta, Jain and Agrawal [5]

(i) $|\arg(z)| < \frac{1}{2}\Omega\pi$ and $\Omega > 0$

(ii) $|\arg(z)| = \frac{1}{2}\Omega\pi$ and $\Omega \ge 0$

and (a) $\varsigma \neq 0$ and contour L is so chosen that $(c\varsigma + \wp + 1) < 0$

(b) $\varsigma = 0$ and $(\wp + 1) < 0$ (6)

where

$$\Omega = \sum_{j=1}^{M} F_j + \sum_{j=1}^{N} \in_j E_j - \sum_{j=M+1}^{Q} F_j \Im_j - \sum_{j=N+1}^{P} E_j \quad \ldots(7)$$

$$\varsigma = \sum_{j=1}^{N} \in_j E_j + \sum_{j=N+1}^{P} E_j - \sum_{j=1}^{M} F_j - \sum_{j=M+1}^{Q} F_j \Im_j \quad \ldots(8)$$

$$\wp = \mathrm{Re}\left(\sum_{j=1}^{M} f_j + \sum_{j=M+1}^{Q} F_j \Im_j - \sum_{j=1}^{N} e_j \in_j - \sum_{j=N+1}^{P} e_j \right) + \frac{1}{2}\left(\sum_{j=1}^{N} \in_j - \sum_{j=M+1}^{Q} \Im_j + P - M - N \right) \quad \ldots(9)$$

It will be appreciated that $S_V^{U_1,\ldots,U_k}[x_1,\ldots,x_k]$ is quite general in nature and reduces to several named multivariable polynomials namely multivariable versions of hypergeometric polynomials, Lauricella polynomials, Jacobi polynomials, Bessel polynomials, Hermite polynomials [1, p.164-168].Evidently the case k =1 of the polynomials (3) would give rise to the general class of polynomials S_V^U introduced by Srivastava [10, p.l, Eq.(1)] and defined in the following manner

$$S_V^U[x] = \sum_{R=0}^{[V/U]} (-V)_{UR} A_{V,R} \frac{x^R}{R!}, \quad V = 0,1,2,\ldots \tag{10}$$

where U is an arbitrary positive integer and the coefficients $A_{V,R}$ (V, R $\geq$ 0) are arbitrary constants, real or complex. The above polynomial is quite general in nature and reduces to several classical polynomials [1 , p.156-163].

Also, the $\overline{H}$-function is very general in nature and its particular cases show a number of important special functions [6, p.159,160].

Our method of solution of integral equation (1) with kernel u(x) given by (2) would depend on the theory of Mellin transform defined by

$$F(s) = M\{f(x);s\} = \int_0^\infty x^{s-1} f(x)\,dx \tag{11}$$

provided that the integral exists.

2. MELLIN TRANSFORM OF U(X)

In order to solve the integral equation (1), we would require the following result contained in

Lemma 2.1 : Let U(s) = M {u(x) ; s} , where u(x) is defined by (2), then

$$U(s)=\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V;R_1,\ldots,R_k)\frac{z_1^{R_1}}{R_1!}\cdots\frac{z_k^{R_k}}{R_k!}$$

$$\lambda^{-1}\ t^{(s+\sum_{i=1}^{k}\rho_iR_i)/\lambda}\ \bar{\phi}\left(\frac{s+\sum_{i=1}^{k}\rho_iR_i}{\lambda}\right) \tag{12}$$

provided that

$$\max_{1\le j\le N}\left[\operatorname{Re}\left\{\in_j\left(\frac{e_j-1}{E_j}\right)\right\}\right]<\operatorname{Re}\left(\frac{s+\sum_{i=1}^{k}\rho_iR_i}{\lambda}\right)<\min_{1\le j\le M}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right]$$

Proof : We have

$$U(s)=M\{u(x);s\}=M\left\{S_V^{U_1,\ldots,U_k}[z_1x^{\rho_1},\ldots,z_kx^{\rho_k}]\,\bar{H}_{P,Q}^{M,N}[t\,x^{-\lambda}]\right\}$$

$$=\int_0^{\infty}x^{s-1}S_V^{U_1,\ldots,U_k}[z_1x^{\rho_1},\ldots,z_kx^{\rho_k}]\,\bar{H}_{P,Q}^{M,N}[t\,x^{-\lambda}]\,dx \tag{13}$$

Expressing the general class of multivariable polynomials $S_V^{U_1,\ldots,U_k}$ in the series form defined by (3) and changing the order of summation and integration (which is permissible under the conditions stated), we get

$$U(s)=\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V;R_1,\ldots,R_k)\frac{z_1^{R_1}}{R_1!}\cdots\frac{z_k^{R_k}}{R_k!}$$

$$M\left\{x^{\sum_{i=1}^{k}\rho_iR_i}\ \bar{H}_{P,Q}^{M,N}[t\,x^{-\lambda}]\ ;\ s\right\} \tag{14}$$

Now, applying the known formulae[3, p.307, Eq.(8)]

$$M\{x^{\mu}f(zx^{-h})\,;s\}=h^{-1}z^{\frac{s+\mu}{h}}\ F\left(-\frac{s+\mu}{h}\right) \tag{15}$$

and [9,p.114, Eq(4.1)]

$$M\left\{ \overline{H}_{P,Q}^{M,N}[x]\,;s \right\} = \overline{\phi}(-s) \tag{16}$$

provided that $\Omega > 0$, $|\arg(x)| < \frac{1}{2}\Omega\pi$ and

$$-\min_{1\le j\le M}\left[\mathrm{Re}\left(\frac{f_j}{F_j}\right)\right] < \mathrm{Re}(s) < \min_{1\le j\le N}\left[\mathrm{Re}\left\{\in_j\left(\frac{1-e_j}{E_j}\right)\right\}\right]$$

in (14), we arrive at the desired result(12).

3. SOLUTION OF THE INTEGRAL EQUATION

The solution of the Fredohlm type integral equation (1) is given in the following theorem

Theorem 3.1 : Let the Mellin transform F(s) , G(s) and U(s) $\neq 0$ of the functions f(x), g(x) and u(x) defined by (2) exist and are analytic in some infinite strip $s_1 < \mathrm{Re}(s) < s_2$ of complex s-plane. Also suppose that for a fixed $c \in (s_1, s_2), u^*(x)$ is defined by

$$u^*(x) = M^{-1}\{U^*(s)\,;\,x\} = \frac{1}{2\pi\omega}\int_{c-\omega\infty}^{c+\omega\infty} x^{-s}\,U^*(s)\,ds \tag{17}$$

where

$$U^*(s) = \left\{ \frac{\mu^{\ell}\,\Gamma(-s/\mu)}{\Gamma[-(s+\mu\ell)/\mu]} \sum_{R_1,\dots,R_k=0}^{\sum_{i=1}^{k} U_iR_i \le V} (-V)_{\sum_{i=1}^{k} U_iR_i}\, A(V;R_1,\dots,R_k)\frac{z_1^{R_1}}{R_1!}\cdots\frac{z_k^{R_k}}{R_k!} \right.$$

$$\left. \lambda^{-1}\, t^{(s+\mu\ell+\gamma+\sum_{i=1}^{k}\rho_iR_i)/\lambda}\;\overline{\phi}\left(\frac{s+\mu\ell+\gamma+\sum_{i=1}^{k}\rho_iR_i}{\lambda}\right)^{-1} \right\} \tag{18}$$

provided that the following conditions are satisfied

(i) $|\arg(t)| < \frac{1}{2}\Omega\pi$ and $\Omega > 0$

(ii) $|\arg(t)| = \frac{1}{2}\Omega\pi$ and $\Omega \ge 0$

and (a) $\varsigma \neq 0$ and contour L is so chosen that $(c\varsigma + \wp + 1) < 0$

(b) $\varsigma = 0$ and $(\wp + 1) < 0$

where

$$\Omega = \sum_{j=1}^{M} F_j + \sum_{j=1}^{N} \in_j E_j - \sum_{j=M+1}^{Q} F_j \Im_j - \sum_{j=N+1}^{P} E_j$$

$$\varsigma = \sum_{j=1}^{N} \in_j E_j + \sum_{j=N+1}^{P} E_j - \sum_{j=1}^{M} F_j - \sum_{j=M+1}^{Q} F_j \Im_j$$

$$\wp = \mathrm{Re}\left(\sum_{j=1}^{M} f_j + \sum_{j=M+1}^{Q} F_j \Im_j - \sum_{j=1}^{N} e_j \in_j - \sum_{j=N+1}^{P} e_j \right) + \frac{1}{2}\left(\sum_{j=1}^{N} \in_j - \sum_{j=M+1}^{Q} \Im_j + P - M - N \right)$$

(iii) $\mu \neq 0, \ell$ is a non –negative integer and min $\{\rho_1, \ldots, \rho_k, \lambda\} > 0$

(iv)

$$\max_{1 \le j \le N} \left[\mathrm{Re}\left\{ \in_j \left(\frac{e_j - 1}{E_j} \right) \right\} \right] < \mathrm{Re}\left(\frac{s + \mu\ell + \gamma + \sum_{i=1}^{k} \rho_i R_i}{\lambda} \right) < \min_{1 \le j \le M} \left[\mathrm{Re}\left(\frac{f_j}{F_j} \right) \right]$$

Then, the integral equation (1) has its solution given by

$$f(x) = x^{-\mu\ell-\gamma} \int_0^\infty y^{-1} u^*\left(\frac{x}{y} \right) \left(y^{\mu+1} D_y \right)^{\ell} \left[y^{\gamma} g(y) \right] dy \qquad (19)$$

provided that the integral on the right hand side of (19) exists.

Proof: To prove the theorem, take the Mellin transform on integral equation (1) and apply the convolution theorem for Mellin transform[3], we get

$$U(s)\,F(s) = G(s) \qquad (20)$$

where U(s) is given by (12) and F(s) and G(s) are Mellin transforms of f(x) and g(x) respectively.

Now replacing s in (20) by $s + \mu\ell + \gamma$ ($\mu \neq 0$ and ℓ is non- negative integer), we have

$$F(s + \mu\ell + \gamma) = U^*(s)\ \mu^{\ell} \left(-\frac{s + \mu\ell}{\mu} \right)_{\ell} G(s + \mu\ell + \gamma) \qquad (21)$$

and using the formula [13]

$$M\left\{ \left(x^{\ell+1} D_x \right)^n f(x)\,;\, s \right\} = \ell^n \left(-\frac{s + \ell n}{\ell} \right)_n F(s + \ell n) \qquad (22)$$

($\ell \neq 0$, n is a non-negative integer)

we get

$$F(s+\mu\ell+\gamma)= U^{*}(s)\ M\left\{\left(y^{\mu+1} D_y\right)^{\ell} \left[y^{\gamma} g(y)\right];s\right\} \tag{23}$$

Again using the elementary result

$$M\left\{x^{\mu} f(x):s\right\} = F(s+\mu) \tag{24}$$

and well known convolution theorem for Mellin transform in (23), we obtain

$$M\left\{x^{\mu\ell+\gamma} f(x);s\right\}=M\left\{\int_0^{\infty} y^{-1} u^{*}\left(\frac{x}{y}\right)\left(y^{\mu+1} D_y\right)^{\ell} \left[y^{\gamma} g(y)\right]dy;s\right\} \tag{25}$$

Inverting both sides of (25) by using well known Mellin inversion theorem, we arrive at the required result(19).

4. SPECIAL CASES

Corollary 4.1 : Reducing general class of multivariable polynomials to unity and the $\overline{H}$-function to generalized Reimann zeta function $\phi(z,p,\eta)$ [6, p.159, Eq.(5)] in the integral equation (1) ,it takes the following form

$$\int_0^{\infty} y^{-1} \phi\left[t\left(\frac{x}{y}\right)^{-\lambda} ,p,\eta \right] f(y)\,dy = g(x) \tag{26}$$

and (18) reduces to

$$U_1^{*}(s) = \frac{\Gamma[-(s+\mu\ell)/\mu]}{\Gamma(-s/\mu)}\ \mu^{-\ell}\ \lambda(-t)^{-(s+\mu\ell+\gamma)/\lambda}$$

$$\frac{\Gamma[1+\eta+(s+\mu\ell+\gamma)/\lambda]^{p}}{\Gamma[-(s+\mu\ell+\gamma)/\lambda]\ \Gamma[1+(s+\mu\ell+\gamma)/\lambda]\ \Gamma[\eta+(s+\mu\ell+\gamma)/\lambda]^{p}} \tag{27}$$

Further,

$$u_1^{*}(x) = M^{-1}\left\{U_1^{*}(s)\,;x\right\}$$

$$= \frac{1}{2\pi\omega}\int_{c-\omega\infty}^{c+\omega\infty} x^{-s}\ U_1^{*}(s)\ ds$$

$$= \frac{\lambda(-t)^{-(\mu\ell+\gamma)/\lambda}\ \mu^{-\ell}}{2\pi\,\omega}\int_{c-\omega\infty}^{c+\omega\infty} (-xt^{1/\lambda})^{-s}\ \frac{\Gamma(-\ell-s/\mu)}{\Gamma(-s/\mu)}$$

$$\frac{\Gamma[1+\eta+(s+\mu\ell+\gamma)/\lambda]^{p}}{\Gamma[-(s+\mu\ell+\gamma)/\lambda]\ \Gamma[1+(s+\mu\ell+\gamma)/\lambda]\ \Gamma[\eta+(s+\mu\ell+\gamma)/\lambda]^{p}}\ ds \tag{28}$$

Now writing the contour integral occurring in(28) in terms of $\overline{H}$-function, we get

$$u_1^*(x) = \lambda(-t)^{-(\mu\ell+\gamma)/\lambda}\,\mu^{-\ell}$$

$$\overline{H}_{3,3}^{1,1}\left[-\frac{1}{xt^{1/\lambda}}\left|\begin{matrix}(-\eta-(\mu\ell+\gamma)/\lambda,1/\lambda;p),\ (0,1/\mu),(-(\mu\ell+\gamma)/\lambda,1/\lambda)\\(-\ell,1/\mu),(-(\mu\ell+\gamma)/\lambda,1/\lambda;1),(1-\eta-(\mu\ell+\gamma)/\lambda,1/\lambda;p)\end{matrix}\right.\right]$$

…(29)

Thus, the integral equation (26) has its solution given by

$$f(x) = x^{-\mu\ell-\gamma}\int_0^\infty y^{-1}u_1^*\left(\frac{x}{y}\right)\left(y^{\mu+1}D_y\right)^{\ell}\left[y^{\gamma}g(y)\right]dy \qquad (30)$$

where $u_1^*(x)$ is given by (29) and the conditions of validity follow easily from those mentioned with the theorem.

Corollary 4.2 : Reducing multivariable polynomial $S_V^{U_1,\dots,U_k}$ to general class of polynomial S_V^U in the theorem, we easily observe that the integral equation

$$\int_0^\infty y^{-1}u_2\left(\frac{x}{y}\right)f(y)dy = g(x) \qquad (31)$$

where

$$u_2(x) = S_V^U[zx^{\rho}]\ \overline{H}_{P,Q}^{M,N}[tx^{-\lambda}] \qquad (32)$$

has its solution given by

$$f(x) = x^{-\mu\ell-\gamma}\int_0^\infty y^{-1}u_2^*\left(\frac{x}{y}\right)\left(y^{\mu+1}D_y\right)^{\ell}\left[y^{\gamma}g(y)\right]dy \qquad (33)$$

provided that integral in (33) exists , $u_2^*(x)$ is the Mellin inverse transform of

$$U_2^*(s) = \left\{\frac{\mu^{\ell}\,\Gamma(-s/\mu)}{\Gamma[-(s+\mu\ell)/\mu]}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}\,A(V,R)\,z^R}{R!}\right.$$

$$\left.\lambda^{-1}\,t^{(s+\mu\ell+\gamma+\rho R)/\lambda}\,\overline{\phi}\left(\frac{s+\mu\ell+\gamma+\rho R}{\lambda}\right)\right\}^{-1} \qquad (34)$$

and the set of conditions of the theorem modified appropriately are satisfied.

If we reduce $\overline{H}$-function to H-function in (32), we get the solution of integral equation in essence the same as considered by Goyal and Mukherjee [4, p.194, Eq.(33)].

Corollary 4.3 : Further, if in corollary(2) we reduce general polynomials $S_V^U[x]$ to Lagurre polynomial $L_V^{\alpha}(x)$ [1, p.158, Eq.(A.8)] and the $\overline{H}$-function to generalized Wright Bessel function $\overline{J}_{\lambda'}^{\nu',\mu'}(x)$ [6,p.160,Eq.(8)] then the integral equation

$$\int_0^\infty y^{-1}L_V^{\alpha}[zx^{\rho}]\ \overline{J}_{\lambda'}^{\nu',\mu'}[tx^{-\lambda}]\,f(y)dy = g(x) \qquad (35)$$

has the solution given by

$$f(x) = x^{-\mu\ell-\gamma} \int_0^{\infty} y^{-1} u_3^*\left(\frac{x}{y}\right)\left(y^{\mu+1} D_y\right)^{\ell} \left[y^{\gamma} g(y)\right]dy \qquad (36)$$

provided that the integral in (36) exists and $\overset{*}{u}_3(x)$ is the Mellin inverse transform of

$$U_3^*(s) = \left\{ \frac{\mu^{\ell}\,\Gamma(-s/\mu)}{\Gamma[-(s+\mu\ell)/\mu]} \sum_{R=0}^{[V]} \binom{V+\alpha}{V} \frac{(-V)_R}{(\alpha+1)_R} \frac{z^R}{R!} \right.$$

$$\left. \lambda^{-1}\, t^{(s+\mu\ell+\gamma+\rho R)/\lambda} \frac{\Gamma[-(s+\mu\ell+\gamma+\rho R)/\lambda]}{\Gamma[1+\lambda'+\nu'(s+\mu\ell+\gamma+\rho R)/\lambda]^{\mu'}} \right\}^{-1}$$

(37)

where the conditions modified appropriately mentioned with corollary 2 are satisfied.

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