Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), **5** (2009), No. 2

A COMPARATIVE STUDY OF SIMULATED VEHICLE ROUTINE PROBLEM AND LINGO SOFTWARE SOLUTION IN LPG DISTRIBUTION SYSTEM

DR. K.K. KAANODIA AND MOHD RIZWANULLAH

Abstract

This paper concerned with the single commodity flow problems on the undirected graph, the exact demand is known and there is single distribution system. But the problem can be further extending to multicommodity flow with more than one supply centre. The objective is to find the optimal rout to supply the commodity to all distribution centre with known demand so as to minimize the cost (ultimately minimize the time). For this we have applied two methods (i) Theoretical Procedure of Traveling Salesman Problem with the help of LINGO Software using of real data and (ii) Simulation techniques by generating the random nos. according to demand. This paper under the comparative study, we find that simulated vehicle routine problem gives the efficient and better solution if the problem consist several nodes (supply centres) with the known distances even the graph is undirected.

Mathematics Subject Classification 2000: 05C85
Additional Key Words and Phrases: Optimization, Tours in Network, Simulation, Traveling Salesman, LINGO, Random numbers, Network Flow, shortest Path.

---

1. INTRODUCTION

Introduction

Tours in network basically concerned with traveling at each node at once with traveling minimum distance and minimum time; where each node is connected by and return to his/her starting point. This problem is not faced by postman only; anyone who is planning a series of deliveries or collection along a number of streets is likely to be intersected in the shortest tour which takes all these streets. Refuse collection vehicles have to follow routes which pass along each street at least once; so do highway inspectors and (by broadening the area of street) inspectors of pipelines and power cables, milk roundsmen and meter readers, traffic wardens and police patrols all face an essential similar types of problem.

The problem can be readily transformed into a network optimization problem; streets can be transformed into arcs of a network, their junctions into nodes, and the distance traveled by the postman in making deliveries along a street into the length associated with that arc. So for a network (N,A,D), the problem is that of identifying the circuit which traverses each arc at least once, and whose total distance is minimum. This problem is sometimes referred as “The Chinese Postman Problem” (Kwan, M-K [27]).

1.1 The Traveling Salesman Problem

Unlike postman, traveling salesman usually concentrates their name on specific towns and locations, rather than the roads and streets which link them. The salesman wishes to visit his client s once, and to plan his route so as to minimize the total distance traveled. At each visit, he has a choice of which one to visit next. Yet, he still wishes to returns to his starting point at the end of his travels.

This problem is not confined to salesman planning calls on clients; drivers of delivery vehicles who have to visit a number of customers in several localities may also wish to minimize their traveling distance, or their travel time. So many operators of vehicles, such as bulk milk Lorries, which make a tour of collection points. In these practical areas, there are other constraints, but nonetheless, the traveling salesman problem frequently forms a part of the algorithm which is used for finding the optimal route for the vehicles. In industrial problem, it is desirable to plan a cyclic schedule of production on a single machine, making one product, changing to another, then another, and eventually returning to the first. The change-over times between different products may mean that some orders for the cycle are quicker than others, and the algorithm for the traveling salesman problem can be used for finding the best.

1.2 LINGO, The Software Used:

LINGO is computer software which can solve both linear as well as non-linear programming problems. The main purpose of using LINGO is to allow its user to input a model formulation, and in return to get the solution. The software assesses the correctness or appropriateness of the formulation and repeats the process. The most powerful feature of LINGO is its ability to model a large system. It has three solvers, which solves different types of models. (i) Direct Solver, (ii) Linear Solver, (iii) Branch-and-bound Solver.

Examining the Solution:

1.3 A Case Study of the Distribution of LPG to the Hall/Hostels in a Campus:

The campus of Aligarh Muslim University is spreaded in 467.6 hectares of land. It comprises of 88 Departments of Studies. There are 14 residential halls for students each hall is constituted of more than one hostel. The hall administration providing two times meal and breakfast to the students in their respective dining hall. A dining hall of a residential hall has a kitchen with good cooking facilities including bio gas stoves. To supply the cooking gas (LPG cylinder) to these 12 kitchens, there is a central AMU Warehouse at some distance from the campus (see AMU Map). The distance among the halls are given in the distance square matrix:

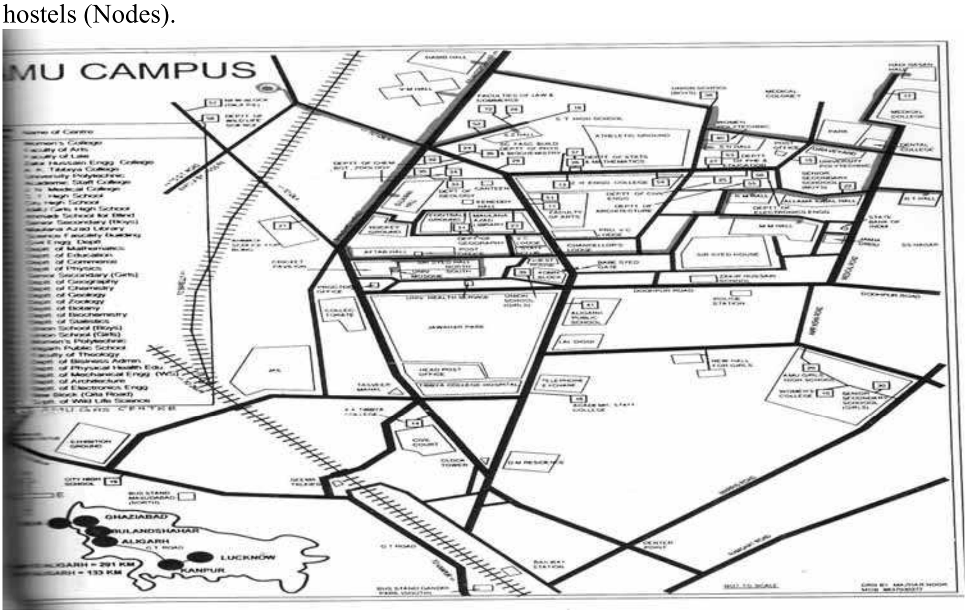
Table- 1 (In meters)

	1	2	3	4	5	67	8	9	10	11
DIST=	AMUCG	SZ	VM	MH	SN	SUL	AF	SS	RM	AI
AMUCG	0	550	635	775	1050	665	930	1080	950	1180
SZ	550	0	285	425	550	315	580	730	600	830
VM	635	285	0	140	490	400	665	815	685	915
MH	775	425	140	0	350	540	805	955	825	1055
SN	1050	550	490	350	0	665	930	1080	330	560
SUL	665	315	400	540	665	0	265	415	715	945
AF	930	580	665	805	930	265	0	150	620	850
SS	1080	730	815	955	1080	415	150	0	690	690

RM	950	600	685	825	330	715	620	690	0	230
AI	1180	830	915	1055	560	945	850	690	230	0
MM	1170	820	905	1045	550	935	620	470	220	220
NT	1470	1120	1205	1345	850	1235	1140	1020	520	290
MH	1560	1210	1050	910	660	1125	1280	1160	660	430

12	13	14
MM	NT	HH
170	1470	1560
820	1120	1210
905	1205	1050
1045	1345	910
550	850	660
935	1235	1125
620	1140	1180
470	1020	1160
220	520	660
220	290	430
0	550	650
550	0	540
650	540	0;

AMUCG: AMU Gas Centre (Warehouse, SZ, VM, …………. Etc. are the name of hostels (Nodes).



LINGO Programe

```
SETS:
        CENTRE / 1..13/: U; !U(I)= SEQUENCE NO. OF LOCATIONS;
        !LVL(I)=level of CENTRE I,U( 1)=0;
        LINK(CENTRE,CENTRE):
        DIST,!The distance matrix;
        X;!X(I,J)=1 if we use link I,J;
```

```
ENDSETS
DATA:   ! Distance matrix need not be symmetric;
    ! However, CENTRE 1 is base of the tree;
!HALL = AMUGC SZ  VM  MH   SN SUL   AF    SS    RM    AI
      MM    NT    NH;
!CENTRE 1 IS BASE;

DIST =
 0    550  635  775  1050 665  930  1080 950  1180 1170 1470 1560
 550  0    285  425  550  315  580  730  600  830  820  1120 1210
 635  285  0    140  490  400  665  815  685  915  905  1205 1050
 775  425  140  0    350  540  805  955  825  1055 1045 1345 910
 1050 550  490  350  0    665  930  1080 330  560  550  850  660
 665  315  400  540  665  0    265  415  715  945  935  1235 1125
 930  580  665  805  930  265  0    150  620  850  620  1140 1180
 1080 730  815  955  1080 415  150  0    690  690  470  1020 1160
 950  600  685  825  330  715  620  690  0    230  220  520  660
 1180 830  915  1055 560  945  850  690  230  0    220  290  430
 1170 820  905  1045 550  935  620  470  220  220  0    550  650
 1470 1120 1205 1345 850  1235 1140 1020 520  290  550  0    540
 1560 1210 1050 910  660  1125 1280 1160 660  430  650  540  0;

ENDDATA
  !The model size: Warning, may be slow for N>=8;
      N = @SIZE(CENTRE);
  !Minimize total distance of the links;
      MIN = @SUM(LINK:DIST*X);
  !For city K, except the base, ... ;
      @FOR(CENTRE(K):
  !It must be entered;
      @SUM(CENTRE(I)|I #NE# K:X(I,K))=1;
  !If there are 2 disjoint tours from 1 CENTRE to
      another, we can remove a link without breaking
      connections. Note: These are not very powerful
      for large problems;
      @SUM(CENTRE(J)|J #NE# K:X(K,J))=1;
      @FOR(CENTRE(J)|J #GT# 1 #AND# J#NE# K:
      U(J)>= U(K)+ X(K,J)-
      (N-2)*(1-X(K,J))+
      (N-3)*X(J,K));
);
  !Make the X's 0/1;
      @FOR(LINK:@BIN(X));
```

```
  !The level of a CENTRE except the base is at least
    1 but no more than N-1, and is 1 if it links to
    the base;
      @FOR(CENTRE(K)|K #GT# 1:
      U(K)<= N-1-(N-2)*X(1,K);
      U(K)>= 1+(N-2)*X(1,K)
);
END
```

Solution of the Problem:

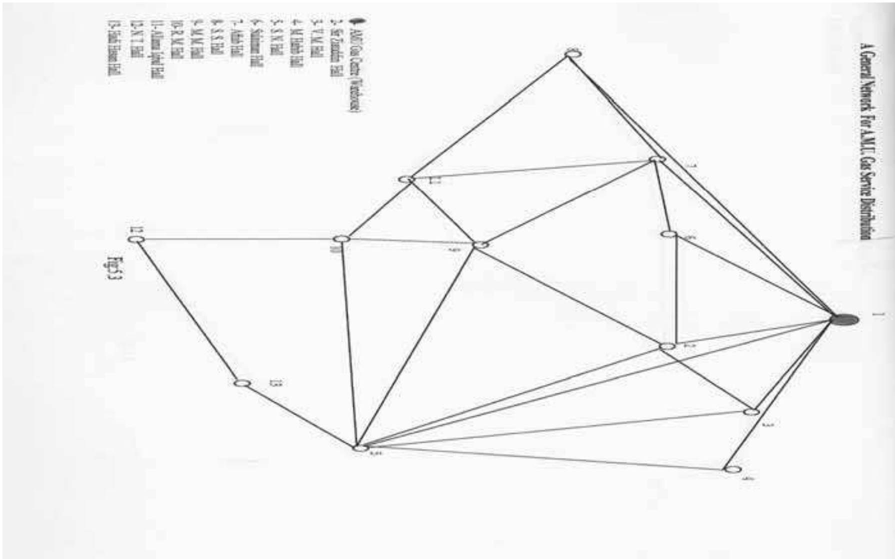
This problem is formulated as Traveling Salesman Problem, after solving the problem with LINGO software, have obtained the optimum rout for the distribution of LPG. The rout obtained through this technique minimizes the time as well as cost of traveling. The optimum route is 1 – 2 –3 – 4 –5 –6 –7 – 8 – 9 – 10 – 11 – 12 – 13 – 1 and the optimum (minimum) distance is 4815.000 meters = 4.815 Km. (Iterative Computational Result in Appendix-I)

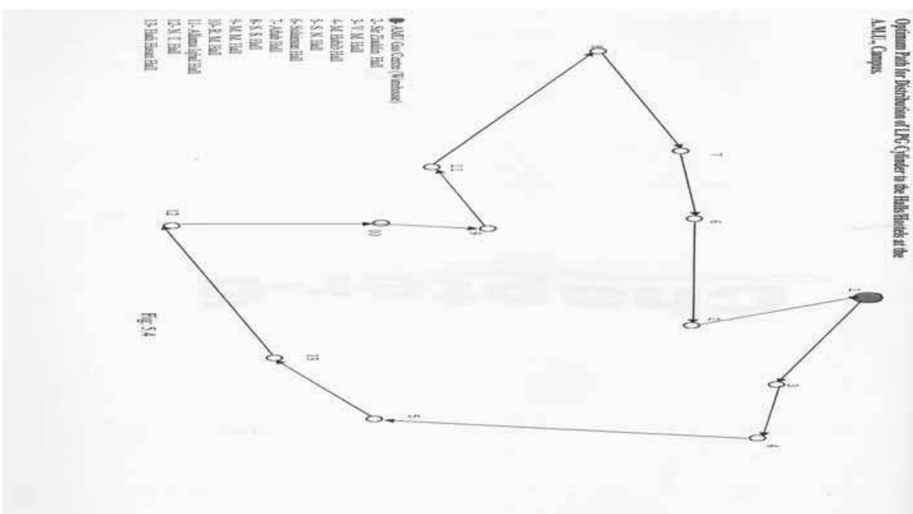
In a LINGO solution report, we'll find a reduced cost figure for each variable. There are two valid, equivalent interpretations of a reduced cost. First, we may interpret a variable's reduced cost as the amount that the objective coefficient of the variable would have to improve before it would become profitable to give the variable in question a positive value in the optimal solution. A variable in the optimal solution, automatically has a reduced cost of zero. Second, the reduced cost of a variable may be interpreted as the amount of penalty we would have to pay to introduce one unit of that variable into the solution.

The Slack or Surplus column in a LINGO solution report tells us how close we are to satisfying a constraint as an equality. This quantity, on less-than-or-equal-to (£) constraints, is generally referred to as slack. On greater-than-or-equal-to (³) constraints, this quantity is called a surplus.

If a constraint is exactly satisfied as equality, the slack or surplus value will be zero. If a constraint is violated, as in an infeasible solution, the slack or surplus value will be negative. Knowing this can help us to find the violated constraints in an infeasible model—a model for which there doesn't exist a set of variable values that simultaneously satisfies all constraints. Non-binding constraints, constraints with a slack or surplus value greater than zero, will have positive, nonzero values in this column.

The LINGO solution report also gives a dual price figure for each constraint. We can interpret the dual price as the amount that the objective would improve as the right-hand side, or constant term, of the constraint is increased by one unit. Dual prices are sometimes called shadow prices, because they tell us how much we should be willing to pay for additional units of a resource.





2. Solution of the Problem of Distribution of LPG Cylinders through Simulation

Simulation is a numerical technique for conducting an experiment on digital computers which involve certain type of mathematical or logical models that describe the behaviour of the problems at extended period of time. It is an indispensable way of solving problems. The Simulation Techniques can be used to established a proper balance between waiting time of customers and idle time of the service facility, to determine the probability distribution of the input and output functions from the past data and run the inventory system artificially by generating the future observations on the assumption of the same distribution. This technique can further be extended to solve the network problem based on investment and budgeting etc.

The problem discussed above (i.e. Distribution of LPG Cylinders in a Campus) is solved with the new method (Simulation Techniques). There are three means of transport in AMU Gas Service Centre (i.e. Tata 407, Bicycle, and 3 Wheelers), they are delivering gas cylinders to different halls in varying duration of time shown in Table-2, gives the distribution route under the column of activity. Next column in the table provides the respective duration of time taken by the three means of transport for each route. For the three durations of time corresponding to one activity, the probability is computed by dividing the individual time duration by the total of the three timings (upto two significant decimal places). These probabilities are given under the column probability of Table-2.

The expected time = $\sum_{i=1}^{3} p_i x_i$ duration is computed for each activity. Cumulative probabilities for each activity are also shown in Table. Tag numbers are computed as follows with the assumption of first tag no. as 00. End of the Tag no. = 100 * Cumulative Probability -1.

Now, for the above problem, we simulate the duration of delivering in 10 times (using the random numbers) and estimated average duration of delivering the cylinders to various halls.

Table-2

Activity	Time (in Minute) x_i	Probability p_i	Expected Duration $\sum_{i=1}^{3} p_i x_i$	Cumulative Probability Distribution	Tag Numbers
1-3	28	0.28		0.28	00-27
	40	0.41	33.54	0.69	28-68
	30	0.31		1.00	69-99
3-4	10	0.27		0.27	00-26
	15	0.41	12.69	0.68	27-67
	12	0.32		1.00	68-99
4-5	20	0.27		0.27	00-26
	30	0.40	25.65	0.67	27-66
	25	0.33		1.00	67-99
5-13	30	0.29		0.29	00-28
	42	0.40	44.12	0.69	29-68
	32	0.31		1.00	69-99
13-12	25	0.28		0.28	00-27
	35	0.39	37.55	0.67	28-66
	30	0.33		1.00	67-99
12-10	12	0.27		0.27	00-26
	18	0.40	15.39	0.67	27-66
	15	0.33		1.00	67-99
10-9	10	0.26		0.26	00-25
	16	0.41	13.45	0.67	26-66
	13	0.33		1.00	67-99
9-11	10	0.26		0.26	00-25

	15	0.39	12.87	0.69	26-64
	13	0.34		1.00	65-99
11-8	22	0.26		0.26	00-25
	33	0.40	28.44	0.66	26-65
	28	0.34		1.00	66-99
8-7	11	0.28		0.28	00-27
	16	0.40	13.64	0.68	28-67
	13	0.32		1.00	68-99
7-6	11	0.26		0.26	00-25
	17	0.41	14.45	0.67	26-66
	14	0.33		1.00	67-99
6-2	18	0.26		0.26	00-25
	28	0.41	23.75	0.67	26-66
	23	0.33		1.00	67-99
2-1	26	0.28		0.28	00-27
	36	0.39	31.55	0.67	28-66
	31	0.33		1.00	67-99

The simulated time for delivering the LPG cylinders through the network with the time duration given in Table-2 are computed. The first column of Table-3 has serial number of ten iteration s of simulations. The next thirteen columns contain the Tag no. (RD) picked up from Random Number Table and their corresponding time duration (D) for each activity. For instance for activity 1-3 the random no. is 17 which fall s in the range of 00 -27 of Tag no. of Table-2 with corresponding time duration 28 minutes, average duration time, in minutes is computed for all the thirteen columns under 'D'. The last row of Table -3 is copied from the fourth column of Table -2 to compute the average duration of time obtained by the simulation and the expected duration time.

The total duration of time for delivering LPG Cylinders is 4.8 hours by the simulation technique while it is 4.87 hours by the theoretical procedure. The precision of estimate obtained by the simulation technique concludes the utility of simulation in traveling salesman problem.

Simulation Run	1-3 RDD		3 – 4 RDD		4 – 5 RDD		5-13 RDD		13-12 RDD		12-10 RDD		10-9 RDD		9-11 RDD		11-8 RDD		8-7 RDD		7-6 RDD		6-2 RDD		2-1 RDD	
1	17	28	77	12	14	20	26	30	11	25	26	12	67	13	73	13	64	33	45	16	87	14	01	18	36	36
2	41	40	71	12	77	25	52	42	87	30	72	15	25	10	47	15	15	22	50	16	27	17	34	28	24	26
3	89	30	39	15	11	20	47	42	24	25	38	18	14	10	99	13	24	22	85	13	27	17	56	28	03	26
4	00	28	74	12	75	25	88	32	18	25	72	15	92	13	87	13	81	28	15	11	12	11	73	23	98	31
5	79	30	80	12	91	25	76	32	79	30	44	18	99	13	87	13	38	33	93	13	22	11	52	28	72	31
6	84	30	07	10	65	30	71	32	47	35	50	18	14	10	14	10	58	33	40	16	74	14	87	23	37	36
7	59	40	42	15	41	30	79	32	51	35	49	18	96	13	70	13	27	33	76	13	45	17	36	28	46	36
8	32	40	40	15	59	30	16	30	45	35	33	18	77	13	15	10	43	33	44	16	20	11	31	28	05	26
9	02	28	04	10	23	20	42	42	29	35	86	15	92	13	62	15	93	328	11	11	60	17	56	28	41	36
10	88	30	43	15	86	25	42	32	38	35	59	18	82	13	68	13	28	33	92	13	95	14	78	23	84	31
Average Duration (in mints.)		32.4		12.8		25.0	79	34.6		31.0		16.5		12.1		12.8		29.1		13.8		14.3		25.5		31.5
Expected duration (in Mints.)		33.54		12.77		25.65		35.42		30.55		15.39		13.45		12.87		28.44		13.64		14.45		23.75		31.50

Table-3 (Iterations)

Total Duration of Delivery Required = 291.82 minutes = 4.8

Total Expected Duration = $\sum_{i=1}^{3} p_i x_i$, where p_i = probability, and x_i = duration

= 292.42 minutes = 4.87 hours

CONCLUSION:

Many types of network optimization problems which in the connection of real life situation can be tackled out by these techniques. The comparative study helps to solve the large routine problem where the real data is not available. Simulation techniques being an important tool for research development can be implemented in such a real life situation.

REFERENCES:

[1] Balakrishnan, A.V. & Lucien, W.N. (1964): "Computing methods in Optimization', Academic Press, New Year.

[2] Bazara, M.S. and Jarvin, J.J. (1977): "Linear Programming and Network Flows ', John Wiley & Sons, New York.

[3] Gupta, S.H. and Lieberman, G.J. (1991): "Introduction to Mathematical Programming", McGraw-Hill Company, New Delhi. Peter, A.S. (1978): "Optimization of Transport Networks", John Wiley & Sons, London.

[4] Bockfeller, R.T. (1984): "Network Flows and Monotropic Optimization", John Wiley & Sons, New York.

[5] Shaghil, M. and Mushtaque, M. (1993): "Network Management Technique", Printwell, Jaipur (India).

[6] Smith, K.S (1982): "Network Optimization Practices: A Computational Guide".

[7] Ahuja R.K., J.B. Orlin, S. Pallottino and M.G. Scutellà, ``Minimum time and minimum cost path problems in street networks with traffic lights", *Transportation Science* 36 (2002) 326-336.

[8] Ahuja R.K., J.B. Orlin, S. Pallottino and M.G. Scutellà, ``Dynamic shortest paths minimizing travel times and costs", *Networks* 41 (2003) 197-205.

[9] Ambrosino D. and M.G. Scutellà, ``Distribution network design: new problems and related models", *European Journal of Operational Research* (2005) to appear.

[10] Bigi G., ``On sufficient second order optimality conditions in multiobjective optimization", *Mathematical Methods of Operations Research*, to appear.

[11] Bigi G., ``Componentwise versus global approaches to nonsmooth multiobjective optimization" *Journal of Industrial and Management Optimization* 1 (2005) 21-32.

[12] Bray, T.A., and C. Witzgall," *Algorithm336: Net flow",* Comm. ACM, Vol. 11 Sept. 1968, 631-632, Corrected in Vol. 12, 1969.

[13] Cappanera P. and A. Frangioni, `Symmetric and asymmetric parallelization of a cost-decomposition algorithm for multi commodity flow problems", *INFORMS Journal On Computing* 15 (2003) 369-384.

[14] Cappanera P. and G. Gallo, ``A multi-commodity flow approach to the crew rostering problem", *Operations Research*, 52 (2004) 583-596.

[15] Cappanera P. and M. G. Scutellà, ``Balanced paths in acyclic networks: tractable cases and related approaches'', *Networks* 45 (2) (2005) 104-111.

[16] Cappanera P. and M. Trubian, ``A local search based heuristic for the demand constrained multidimensional knapsack problem'', *INFORMS Journal On Computing* 17 (2005) 82-98.

[17] Chekuri C., G.P.Oriolo, M.G.Scutellà and F.B.Shepherd, ``Hardness of Robust Network Design'', Proceedings of INOC 2005, Lisbon (2005) 455-461.

[18] Elmaghraby, S.E. "*Some Network Models in Management Science*", Vol. 17, Sept. and Oct. 1970.

[19] Frangioni A., ``About Lagrangian Methods in Integer Optimization'', *Annals of Operations Research* (2005) to appear.

[20] Frangioni A. and B. Gendron, ``0-1 Reformulations of the Network Loading Problem'', Proceedings INOC2005 conference, Lisbon (2005) 38-43.

[21] Frangioni A. and C. Gentile, ``New preconditioners for KKT systems of network flow problems'', *SIAM Journal on Optimization* (2003) to appear.

[22] Frangioni A., A. Lodi and G. Rinaldi, ``Optimizing over Semimetric Polytopes'' in *Integer Programming and Combinatorial Optimization - IPCO 2004*, D. Bienstock and G. Nemhauser eds., Lecture Notes in Computer Science Vol. 3064, Springer-Verlag (2004) 431-443.

[23] Frangioni A. and A. Manca, ``A computational study of cost reoptimization for min cost flow problems'', *INFORMS Journal On Computing* (2005) to appear.

[24] Fulkerson, D.R. "Flow Networks and Combinatorial Operations", *An. Math*, Monthly, Vol. 73, 1966, 115-138.

[25] Gentile G., S. Nguyen and S. Pallottino, ``Route choice on transit networks with on-line information at stops'', TR 03-14, Dipartimento di Informatica, Università di Pisa (2003) submitted to *Transportation Science*.

[26] Henri, Pierreval and Jean, Luc Paris: "From Simulation optimization to Simulation Configuration of the System", Vol. 11, Issue 1, 15 March, 20023. Page 5-19.

[27] Nguyen S., S. Pallottino and F. Malucelli, ``A modeling framework for the passenger assignment on a transport network with time-tables'', *Transportation Science* 35 (2001) 238-249.

[28] Nguyen S., S. Pallottino and M.G. Scutellà, ``A new dual algorithm for shortest path reoptimization'', in (M. Gendreau and P. Marcotte, eds.) *Transportation and Nework Analysis: Current Trends*, Kluwer (2002) 221-235.

[29] Pallottino S. and M.G. Scutellà, ``A new algorithm for reoptimizing shortest paths when the arc costs change'', *Operations Research Letters* 31 (2003) 149-160.

[30] Pallottino S. and R. Tadei (Editors), *Communication and Transportation Network Optimization*, special issue of *Networks* (2003) to appear.

[31] Scutellà M.G., ``An approximation algorithm for computing longest paths'', *European Journal of Operational Research* 148(3) (2003) 584-590.

[32] Scutellà M.G., ``A note on the parametric maximum flow problem and some related reoptimization issues'', submitted to *Annals of Operations Research*.

Dr. K.K. Kaanodia
Reader
Department of Mathematics
BSA (P.G.) College, Mathura.
281004, (U.P.), India.
E-mail: kk.kaanodiya@yahoo.com

Mohd. Rizwanullah
Research Scholar
Department of Mathematics
B.S.A., (P.G.) College, Mathura.
281004, (U.P.), India.
E-mail: rizwansal@yahoo.co.in

Received September 2009

Appendix-I

Iterative Computation of the LPG Distribution System as Traveling Salesman Problem (LINGO Solution)

Global optimal solution found at step:
1239
Objective value:
4815.00
N 13.00000

Variable	Value		
U(1)	1.234568	X(5, 10)	1.000000
U(2)	1.234568	X(5, 11)	1.000000
U(3)	1.234568	X(5, 12)	1.000000
U(4)	1.234568	X(5, 13)	1.000000
U(5)	1.234568	X(6, 1)	1.000000
U(6)	1.234568	X(6, 2)	1.000000
U(7)	1.234568	X(6, 3)	1.000000
U(8)	1.234568	X(6, 4)	1.000000
U(9)	1.234568	X(6, 5)	1.000000
U(10)	1.234568	X(6, 6)	1.000000
U(11)	1.234568	X(6, 7)	1.000000
U(12)	1.234568	X(6, 8)	1.000000
U(13)	1.234568	X(6, 9)	1.000000
DIST(1, 1)	0.000000	X(6, 10)	1.000000
DIST(1, 2)	550.0000	X(6, 11)	1.000000
DIST(1, 3)	635.0000	X(6, 12)	1.000000
DIST(1, 4)	775.0000	X(6, 13)	1.000000
DIST(1, 5)	1050.000	X(7, 1)	1.000000
DIST(1, 6)	665.0000	X(7, 2)	1.000000
DIST(1, 7)	930.0000	X(7, 3)	1.000000
DIST(1, 8)	1080.000	X(7, 4)	1.000000
DIST(1, 9)	950.0000	X(7, 5)	1.000000
DIST(1, 10)	1180.000	X(7, 6)	1.000000
DIST(1, 11)	170.0000	X(7, 7)	1.000000
DIST(1, 12)	1470.000	X(7, 8)	1.000000
DIST(1, 13)	1560.000	X(7, 9)	1.000000
DIST(2, 1)	550.0000	X(7, 10)	1.000000
DIST(2, 2)	0.000000	X(7, 11)	1.000000
		X(7, 12)	1.000000
		X(7, 13)	1.000000

DIST(2, 3)	285.0000	X(8, 1)	1.000000
DIST(2, 4)	425.0000	X(8, 2)	1.000000
DIST(2, 5)	550.0000	X(8, 3)	1.000000
DIST(2, 6)	315.0000	X(8, 4)	1.000000
DIST(2, 7)	580.0000	X(8, 5)	1.000000
DIST(2, 8)	730.0000	X(8, 6)	1.000000
DIST(2, 9)	600.0000	X(8, 7)	1.000000
DIST(2, 10)	830.0000	X(8, 8)	1.000000
DIST(2, 11)	820.0000	X(8, 9)	1.000000
DIST(2, 12)	1120.000	X(8, 10)	1.000000
DIST(2, 13)	1210.000	X(8, 11)	1.000000
DIST(3, 1)	635.0000	X(8, 12)	1.000000
DIST(3, 2)	285.0000	X(8, 13)	1.000000
DIST(3, 3)	0.000000	X(9, 1)	1.000000
DIST(3, 4)	140.0000	X(9, 2)	1.000000
DIST(3, 5)	490.0000	X(9, 3)	1.000000
DIST(3, 6)	400.0000	X(9, 4)	1.000000
DIST(3, 7)	665.0000	X(9, 5)	1.000000
DIST(3, 8)	815.0000	X(9, 6)	1.000000
DIST(3, 9)	685.0000	X(9, 7)	1.000000
DIST(3, 10)	919.0000	X(9, 8)	1.000000
DIST(3, 11)	905.0000	X(9, 9)	1.000000
DIST(3, 12)	1205.000	X(9, 10)	1.000000
DIST(3, 13)	1050.000	X(9, 11)	1.000000
DIST(4, 1)	775.0000	X(9, 12)	1.000000
DIST(4, 2)	425.0000	X(9, 13)	1.000000
DIST(4, 3)	140.0000	X(10, 1)	1.000000
DIST(4, 4)	0.000000	X(10, 2)	1.000000
DIST(4, 5)	350.0000	X(10, 3)	1.000000
DIST(4, 6)	540.0000	X(10, 4)	1.000000
DIST(4, 7)	805.0000	X(10, 5)	1.000000
DIST(4, 8)	955.0000	X(10, 6)	1.000000
DIST(4, 9)	825.0000	X(10, 7)	1.000000
DIST(4, 10)	1055.000	X(10, 8)	1.000000
DIST(4, 11)	1045.000	X(10, 9)	1.000000
DIST(4, 12)	1345.000	X(10, 10)	1.000000
DIST(4, 13)	910.0000	X(10, 11)	1.000000
DIST(5, 1)	1050.000	X(10, 12)	1.000000
DIST(5, 2)	550.0000	X(10, 13)	1.000000
DIST(5, 3)	490.0000	X(11, 1)	1.000000
DIST(5, 4)	350.0000	X(11, 2)	1.000000
DIST(5, 5)	0.000000	X(11, 3)	1.000000
DIST(5, 6)	665.0000	X(11, 4)	1.000000
DIST(5, 7)	930.0000	X(11, 5)	1.000000
DIST(5, 8)	1080.000	X(11, 6)	1.000000
DIST(5, 9)	330.0000	X(11, 7)	1.000000
DIST(5, 10)	560.0000	X(11, 8)	1.000000
DIST(5, 11)	550.0000	X(11, 9)	1.000000
DIST(5, 12)	850.0000	X(11, 10)	1.000000
DIST(5, 13)	660.0000	X(11, 11)	1.000000
DIST(6, 1)	665.0000	X(11, 12)	1.000000
DIST(6, 2)	315.0000	X(11, 13)	1.000000
DIST(6, 3)	400.0000	X(12, 1)	1.000000
DIST(6, 4)	540.0000	X(12, 2)	1.000000
DIST(6, 5)	665.0000	X(12, 3)	1.000000
DIST(6, 6)	0.000000	X(12, 4)	1.000000
DIST(6, 7)	265.0000	X(12, 5)	1.000000
DIST(6, 8)	415.0000	X(12, 6)	1.000000
DIST(6, 9)	715.0000	X(12, 7)	1.000000
DIST(6, 10)	945.0000	X(12, 8)	1.000000
DIST(6, 11)	935.0000	X(12, 9)	1.000000
DIST(6, 12)	1235.000	X(12, 10)	1.000000
DIST(6, 13)	1125.000	X(12, 11)	1.000000
DIST(7, 1)	930.0000	X(12, 12)	1.000000
DIST(7, 2)	580.0000	X(12, 13)	1.000000
DIST(7, 3)	665.0000	X(13, 1)	1.000000
DIST(7, 4)	805.0000	X(13, 2)	1.000000
DIST(7, 5)	930.0000	X(13, 3)	1.000000
DIST(7, 6)	265.0000	X(13, 4)	1.000000
DIST(7, 7)	0.000000	X(13, 5)	1.000000

DIST(7, 8)	150.0000
DIST(7, 9)	620.0000
DIST(7, 10)	850.0000
DIST(7, 11)	620.0000
DIST(7, 12)	1140.000
DIST(7, 13)	1180.000
DIST(8, 1)	1080.000
DIST(8, 2)	730.0000
DIST(8, 3)	815.0000
DIST(8, 4)	955.0000
DIST(8, 5)	1080.000
DIST(8, 6)	415.0000
DIST(8, 7)	150.0000
DIST(8, 8)	0.000000
DIST(8, 9)	690.0000
DIST(8, 10)	690.0000
DIST(8, 11)	470.0000
DIST(8, 12)	1020.000
DIST(8, 13)	1160.000
DIST(9, 1)	950.0000
DIST(9, 2)	600.0000
DIST(9, 3)	685.0000
DIST(9, 4)	825.0000
DIST(9, 5)	330.0000
DIST(9, 6)	715.0000
DIST(9, 7)	620.0000
DIST(9, 8)	690.0000
DIST(9, 9)	0.000000
DIST(9, 10)	230.0000
DIST(9, 11)	220.0000
DIST(9, 12)	520.0000
DIST(9, 13)	660.0000
DIST(10, 1)	1180.000
DIST(10, 2)	830.0000
DIST(10, 3)	915.0000
DIST(10, 4)	1055.000
DIST(10, 5)	560.0000
DIST(10, 6)	945.0000
DIST(10, 7)	850.0000
DIST(10, 8)	690.0000
DIST(10, 9)	230.0000
DIST(10, 10)	0.000000
DIST(10, 11)	220.0000
DIST(10, 12)	290.0000
DIST(10, 13)	430.0000
DIST(11, 1)	1170.000
DIST(11, 2)	820.0000
DIST(11, 3)	905.0000
DIST(11, 4)	1045.000
DIST(11, 5)	550.0000
DIST(11, 6)	935.0000
DIST(11, 7)	620.0000
DIST(11, 8)	470.0000
DIST(11, 9)	220.0000
DIST(11, 10)	220.0000
DIST(11, 11)	0.000000
DIST(11, 12)	550.0000
DIST(11, 13)	650.0000
DIST(12, 1)	1470.000
DIST(12, 2)	1120.000
DIST(12, 3)	1205.000
DIST(12, 4)	1345.000
DIST(12, 5)	850.0000
DIST(12, 6)	1235.000
DIST(12, 7)	1140.000
DIST(12, 8)	1020.000
DIST(12, 9)	520.0000
DIST(12, 10)	290.0000
DIST(12, 11)	550.0000
DIST(12, 12)	0.000000
DIST(12, 13)	540.0000
DIST(13, 1)	1560.000

X(13, 6)	1.000000
X(13, 7)	1.000000
X(13, 8)	1.000000
X(13, 9)	1.000000
X(13, 10)	1.000000
X(13, 11)	1.000000
X(13, 12)	1.000000
X(13, 13)	1.000000

Row	Slack or Surplus
1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000
25	0.000000
26	0.000000
27	0.000000
28	0.000000
29	0.000000
30	0.000000
31	0.000000
32	0.000000
33	0.000000
34	0.000000
35	0.000000
36	0.000000
37	0.000000
38	0.000000
39	0.000000
40	0.000000
41	0.000000
42	0.000000
43	0.000000
44	0.000000
45	0.000000
46	0.000000
47	0.000000
48	0.000000
49	0.000000
50	0.000000
51	0.000000
52	0.000000
53	0.000000
54	0.000000
55	0.000000
56	0.000000
57	0.000000
58	0.000000
59	0.000000
60	0.000000
61	0.000000
62	0.000000

DIST(13, 2)	1210.000	63	0.000000
DIST(13, 3)	1050.000	64	0.000000
DIST(13, 4)	910.0000	65	0.000000
DIST(13, 5)	660.0000	66	0.000000
DIST(13, 6)	1125.000	67	0.000000
DIST(13, 7)	1280.000	68	0.000000
DIST(13, 8)	1160.000	69	0.000000
DIST(13, 9)	660.0000	70	0.000000
DIST(13, 10)	430.0000	71	0.000000
DIST(13, 11)	650.0000	72	0.000000
DIST(13, 12)	540.0000	73	0.000000
DIST(13, 13)	0.000000	74	0.000000
X(1, 1)	1.000000	75	0.000000
X(1, 2)	1.000000	76	0.000000
X(1, 3)	1.000000	77	0.000000
X(1, 4)	1.000000	78	0.000000
X(1, 5)	1.000000	79	0.000000
X(1, 6)	1.000000	80	0.000000
X(1, 7)	1.000000	81	0.000000
X(1, 8)	1.000000	82	0.000000
X(1, 9)	1.000000	83	0.000000
X(1, 10)	1.000000	84	0.000000
X(1, 11)	1.000000	85	0.000000
X(1, 12)	1.000000	86	0.000000
X(1, 13)	1.000000	87	0.000000
X(2, 1)	1.000000	88	0.000000
X(2, 2)	1.000000	89	0.000000
X(2, 3)	1.000000	90	0.000000
X(2, 4)	1.000000	91	0.000000
X(2, 5)	1.000000	92	0.000000
X(2, 6)	1.000000	93	0.000000
X(2, 7)	1.000000	94	0.000000
X(2, 8)	1.000000	95	0.000000
X(2, 9)	1.000000	96	0.000000
X(2, 10)	1.000000	97	0.000000
X(2, 11)	1.000000	98	0.000000
X(2, 12)	1.000000	99	0.000000
X(2, 13)	1.000000	100	0.000000
X(3, 1)	1.000000	101	0.000000
X(3, 2)	1.000000	102	0.000000
X(3, 3)	1.000000	103	0.000000
X(3, 4)	1.000000	104	0.000000
X(3, 5)	1.000000	105	0.000000
X(3, 6)	1.000000	106	0.000000
X(3, 7)	1.000000	107	0.000000
X(3, 8)	1.000000	108	0.000000
X(3, 9)	1.000000	109	0.000000
X(3, 10)	1.000000	110	0.000000
X(3, 11)	1.000000	111	0.000000
X(3, 12)	1.000000	112	0.000000
X(3, 13)	1.000000	113	0.000000
X(4, 1)	1.000000	114	0.000000
X(4, 2)	1.000000	115	0.000000
X(4, 3)	1.000000	116	0.000000
X(4, 4)	1.000000	117	0.000000
X(4, 5)	1.000000	118	0.000000
X(4, 6)	1.000000	119	0.000000
X(4, 7)	1.000000	120	0.000000
X(4, 8)	1.000000	121	0.000000
X(4, 9)	1.000000	122	0.000000
X(4, 10)	1.000000	123	0.000000
X(4, 11)	1.000000	124	0.000000
X(4, 12)	1.000000	125	0.000000
X(4, 13)	1.000000	126	0.000000
X(5, 1)	1.000000	127	0.000000
X(5, 2)	1.000000	128	0.000000
X(5, 3)	1.000000	129	0.000000
X(5, 4)	1.000000	130	0.000000
X(5, 5)	1.000000	131	0.000000
X(5, 6)	1.000000	132	0.000000

X(5, 7)	1.000000	133	0.000000
X(5, 8)	1.000000	134	0.000000
X(5, 9)	1.000000	135	0.000000
		136	0.000000
		137	0.000000
		138	0.000000
		139	0.000000
		140	0.000000
		141	0.000000
		142	0.000000
		143	0.000000
		144	0.000000
		145	0.000000
		146	0.000000
		147	0.000000
		148	0.000000
		149	0.000000
		150	0.000000
		151	0.000000
		152	0.000000
		153	0.000000
		154	0.000000
		155	0.000000
		156	0.000000
		157	0.000000
		158	0.000000
		159	0.000000
		160	0.000000
		161	0.000000
		162	0.000000
		163	0.000000
		164	0.000000
		165	0.000000
		166	0.000000
		167	0.000000
		168	0.000000
		169	0.000000
		170	0.000000
		171	0.000000
		172	0.000000
		173	0.000000
		174	0.000000
		175	0.000000
		176	0.000000
		177	0.000000
		178	0.000000
		179	0.000000
		180	0.000000
		181	0.000000
		182	0.000000
		183	0.000000
		184	0.000000
		185	0.000000
		186	0.000000
		187	0.000000
		188	0.000000
		189	0.000000
		190	0.000000
		191	0.000000
		192	0.000000
		193	0.000000
		194	0.000000
		195	0.000000
		196	0.000000