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A DISCUSSION ON LUMBAR SPINE RECONSTRUCTION: A SEMI STOCHASTIC PROCESSES APPROACH[1]

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Abstract

An understanding of the mechanics of the human spine is pertinent to numerous medical problems. Among these there are pathogenesis of the disc degeneration, effect of vertebral arthrodesis, endplate fractures during pilot ejection, disc replacement and development and correction of spinal deformities. Since mechanical factor plays a significant role in the onset of low back pain, in vitro biomechanical studies may be undertaken to provide a basic understanding of the proportional resistance offered by the various spinal elements in response to external loading. The aim of this paper is to discuss possible 3D-reconstruction of the spine using a combined geometrical, static and statistical model. We also illustrate the deterministic phase of the semi stochastic process by the equilibrium on a simulated data set.

Mathematics Subject Classification 2000: 91B70
Additional Key Words and Phrases: human spine, 3D-reconstruction of the spine, statistical model

1. INTRODUCTION

A human spine – its theoretical and experimental investigation belongs steadily to the serious biomechanical problems. Especially its lumbar part awakes a big attention. It is well-known that human spine can be characterized as a spatially double-curved build-up rod of composite structure. This space rod is simultaneously under flexural, compressive and/or torsional effects. In the pathological (or after injure) state their "admirable" equilibrium state is disturbed. Knowledge of the mechanical response and deformability of the intervertebral discs provides us to predict a degeneration of human spine. From the medical point of view these experiences influence on appropriate selection of medical therapy.

The spinal elements of a motion segment impact the much-needed stability / flexibility to the segment. Their relative contributions in resisting various load types can be estimated by determining the load-displacement behavior of intact specimens.

Although the exact mechanism is permanently not known, epidemiological studies (Damkot 1984, Frymoyer 1983, Kelsey 1984) have shown that mechanical factors of various kinds (inappropriate work habits, poorly designed chairs, repetitive loading

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(flexion and / or twisting) of the spine in an industrial setting) imposed during daily living do play a significant role in the onset of chronic low back pain. Since mechanical factors play a significant role in the onset of low back pain, in vitro biomechanical studies - may be undertaken to provide a basic understanding of the proportional resistance offered by the various spinal elements in response to external loading. However, at the present time, even some of the salient features of the spine mechanical behavior are poorly understood. This is a consequence of the difficulties of (1) determining in vivo mechanical properties and (2) constructing a realistic analytical model. For example overall static force deflection properties of individual intervertebral disc have been measured by Rolander, Nachemson , Brown et al and Markolf among others.

A complete characterization of the material properties of the annulus fibrosis is however an almost insurmountable task, for it is orthotropic and hence at last seven distinct material constants must be determined for a linear analysis. Experimental studies of motion segment mechanical behavior, e.g. Nachemson 1979 show that it can vary widely depended on the age, morphology position and degree of degeneration. Deformation properties of annulus fibrosis, as well as and other components were studied e.g. in Sumec et al. 1996.

One of the first semi-experimental models of the individual disc was developed by Sonnerup.

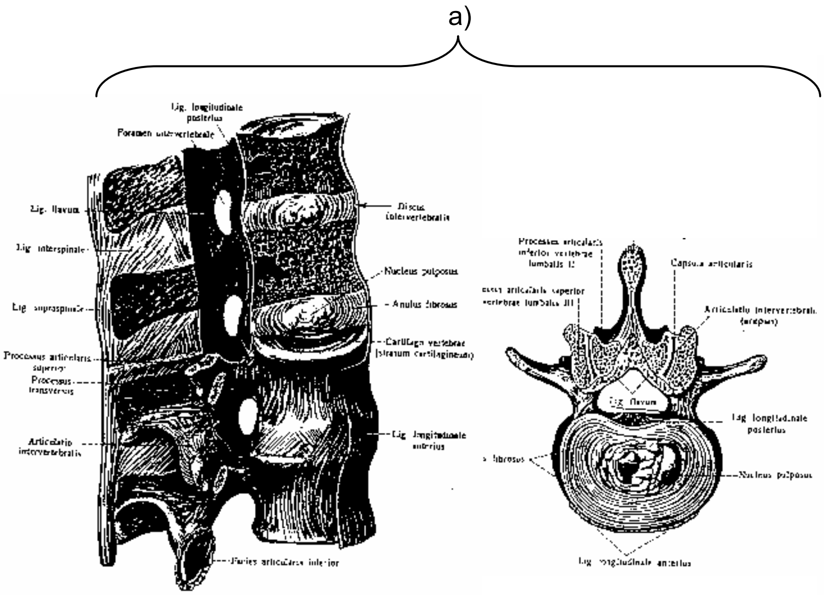
The aim of this contribution is a study of the stress-strain distribution, deflection and stiffness of human spine under axial loads. This work is extend of results and computational procedures published by Sokol and Sumec in 2000.

The paper is organized as follows. In the following section we provide the morphology of human spine. In the 3rd section we provide the methodological discussion on the 3d reconstruction of the himan spine via the semi stochastic process. In the last section we provide the illustrative example of deterministic stage for a semi stochastic process.

2. MORPHOLOGY OF THE HUMAN SPINE

The human spine consists of seven cervicalia vertebrae (C1 - C7), and twelve dorsal vertebrae (T1 - T12), five lumbar vertebrae (L1 - L5) five crucial vertebrae (S1 – S5) and four or five coccygis vertebrae which lead to the one segment. Human spine is not a straight column, but it is curved in the sagittal and frontal plane.

Convex back curvature (kyphosis) in the dorsal and crucial zones is occurred. Convex forward curvature (lordosis) is in the cervical and lumbar zones. The changes of curvatures are continuous, only transition of the lumbar lordosis into crucial kyphosis is folded. The sagittal view on spine is given in Fig. 1b. In the frontal plane the spine is smoothly buckled in side. The intervertebral disc consists of three parts: the annulus fibrosis in the periphery, the cartilage plates above the bellow and the nucleus pulposus in the center. The annulus fibrosis in adult lumbar spine is formed by a series of concentric encircling lamellae. The nucleus pulposus is located nearly in the center of the disc. The water content of the nucleus ranges from 88 per cent at birth to approximately 70 per cent at the age of 77. The scheme of the part of human spine as well as the internal structure of the intervertebra disc is given in the Fig. 1a.



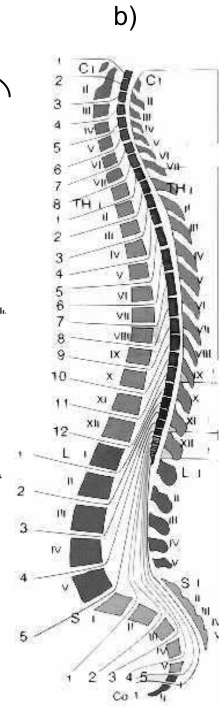


Fig. 1. Scheme of the human spine and anatomy of the human spine segments

3. 3D-RECONSTRUCTION OF THE SPINE USING A COMBINED GEOMETRICAL, STATIC AND STATISTICAL MODEL

Stereoradiographic methods based on anatomical landmarks identification are the only ones providing information on 3D-deformities of the spine in a standing position, but require 24 h for the whole spine, making the method inadequate for clinical routine. Recent publication (Pomero et al.2004) has proposed the semi-automated method based on (1) vertebral body volume reconstruction, (2) definition of a local referential associated to this volume, (3) reliable a priori knowledge of the vertebral shape using eight morphologic descriptors of the vertebral body to estimate, from a multiple linear regression, 21 3D-point coordinates per vertebra, (4) kriging of a 2000 points model with regard to the 21 points. The method was evaluated for vertebral orientation and shape accuracy. Within the approach proposed by Pomero et al.2004. 3D models of the whole spine are obtained within 15 min. Manual vs. semi-automated reconstruction comparison yield similar accuracy regarding the CT-scan references. For vertebrae orientation, results were slightly different from the manual reconstruction method (however an absolute reference is lacking). Two to four hours may be required to reconstruct the whole spine, especially when dealing with scoliotic spines. The purpose of this work is to propose the original method for a fast 3D reconstruction based on an use of a priori knowledge of vertebral and spinal geometry. The problems, which are not answered until now:

1) What is the sufficient number of landmarks to build up (based up on in-vivo observation) a single vertebra?

2) How to design them on the vertebra, if we consider a correlation between the observations?

3) Could we improve a much if we instead 1) and 2) use a massive data set spreading uniformly (space filling design) on the surface of vertebra? To process a massive data set is a computational challenge.

As was observed at Stehlík et al 2008, there are some crucial times when the observation of the patient has to be made. Also there are some crucial points at which we have to measure to obtain sufficient information about the spine behaviour. The sufficiency has been observed in the sense of unbounded Statistical information at these designs and we can conjecture that theoretically we observe the convergence of MLE distributions to Dirac functions, e.g. we obtain the deterministic limit of a sequence of variables with positive variance. Such problems are touching the singular processes. Also the experience of the Medical doctors can support the argument, that medical treatment is always a finite-dimensional spatiotemporal design. For a good reference on a spatial design see Müller 2007. A semi stochastic process (see Ross 1995) is a stochastic process with memory and deterministically described jumps (typically by the ordinary or partial diff. equations). The case of spatially deformed data and its possible generalization via topological approach is discussed in Stehlík 2009. The important issue for a further investigation are cases, when the process becomes semi-Markovian.

The interesting thing in this methodology is, that we can define the important characterization via entropy (neg-information) or energy (relation between information and virtual energy can be found in Stehlík 2007). To see the possibility of growth modeling via Entropy, we can concentrate on diffusion processes over porous medium, for the sake of simplicity with fractal structure. For diffusions in Euclidean spaces R^d we have the continuous jump probability of Gaussian density form $p(x) = A\exp(-x^2/2\sigma^2)$ which is the extremal solution of optimization of Boltzmann Entropy $E_B = -K_B \int p(x)\ln p(x)dx$ under the constrains $\int p(x)dx = 1, \int x^2 p(x)dx = \sigma^2 d,$ where d is the spatial dimension and $x = |x|$. The topological considerations also fort he diffusion processes can be found in Ďurikovič 2005. However, for diffusions in fractals with fractal dimension γ we have the continuous jump probability with Pareto (for $\gamma > 0$ so called stable) tail $p(x) \approx x^{-1-\gamma}$ which is the extremal solution (see Alemany 1994) of optimization of Tsallis Entropy $E_T = -K\frac{1-\int p(x)^q dx}{1-q}$ under the natural constrains $\int p(x)dx = 1, \int x^2 p^q(x)dx = \Omega_d \int x^{d-1}x^2 p^q(x)dx,$ where Ω_d is the total solid angle in the d-dimensional space. For the exact LR testing of the fractal dimension from one sample see Stehlík 2009 and for two pseudoexponentially dependent samples see Filus et al. 2009.

4. ILLUSTRATIVE EXAMPLE OF A DETERMINISTIC STAGE OF THE SEMI-STOCHASTIC PROCESS

While solving stress - strain problem we will start from the energetic principle. It can be shown that all energetic theorems can be derived from the principle of virtual work (virtual translation) or from the principle of complementary virtual work (virtual forces), see Frymoyer 1991.

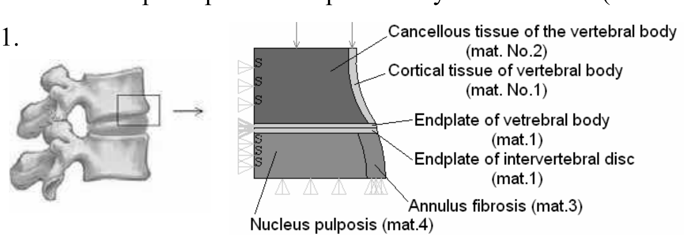


Fig.2. 2D representation of the body of a vertebra and intervertebral disc.

We suppose the body – a vertebra joined with intervertebral disc along whole endplates, see Fig 1. - is continuously deformable for physiologic load values and fulfilled the geometric boundary conditions.

4.1. Mathematical model

In undeformed stage the volume of the domain Ω is V_Ω with boundary S_Ω. The domain consists of subdomains V_i^Ω with boundaries S_i^Ω. In general we suppose, the volume forces $\overline{X}_i$, (i = 1, 2, 3) act inside the domain. The boundary of the domain S_Ω will consist of two parts S_Ω^1, S_Ω^2. On S_Ω^1 the vector of stress $\bar{\sigma}_i^{(v)}$, (i = 1, 2, 3), i.e. forces acting on the area unit with outer normal $v = (v_1, v_2, v_3)$. On S_Ω^2 the displacements $\bar{\mu}_i$ will be given. We suppose the whole body is in equilibrium. We also suppose the $\bar{\mu}_i$ are continuous function with continuous derivatives up to the third order and $\bar{\sigma}_i$ are continuous function with continuous derivatives up to the second order.

For virtual work we can write

$$\int_{V_\Omega}(\sigma_{ij,j} + X_i)\delta u_i dV + \int_{S_\Omega^1}\left(\bar{\sigma}_i^{(v)} - \sigma_{i,j}v_i\right)\delta u_i dS = 0. \tag{1}$$

In the meaning of Gauss theorem we have

$$\int_{V_\Omega}\sigma_{ij,j}\delta u_i dV = \int_{S_\Omega}\sigma_{i,j}v_i\delta u_i dS - \int_{V_\Omega}\sigma_{i,j}(\delta u_i)_{,j}dV. \tag{2}$$

After substitution (2) to (1) we have

$$-\int_{V_\Omega}\sigma_{i,j}(\delta u_i)_{,j}dV + \int_{V_\Omega}\bar{x}_i\delta u_i dV + \int_{S_\Omega^1}\bar{\sigma}_i^{(v)}\delta u_i dS + \int_{S_\Omega^1}\bar{\sigma}_i^{(v)}\delta u_i dS = 0. \tag{3}$$

Furthermore

$\sigma_{ij,j} + X_i = 0$ on Ω - *equilibrium equation*

$\sigma_i^{(v)} = \bar{\sigma}_i^{(v)}$, $\sigma_i^{(v)} = \sigma_{ij}v_i$ on S_Ω^1 - *static boundary conditions*

$u_i = \bar{u}_i$ on S_Ω^2 - *geometric boundary conditions*

Because of symmetry of the stress vector we obtain

$$\int_{V_\Omega}\sigma_{i,j}\varepsilon_{ij}dV - \int_{V_\Omega}\bar{x}_i\delta u_i dV - \int_{S_\Omega^1}\bar{\sigma}_i^{(v)}\delta u_i dS = 0 \tag{4}$$

where

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ is a *small deformation tensor*

Equation (4) represents virtual work principle, even in the case of *finite deformation*, where

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i} + u_{k,j})$, and in physically *nonlinear* case.

Let $\bar{U}$ be the deformation energy density. Then

$$\delta U = \delta\sigma_{ij}\delta\varepsilon_{ij} \quad .$$

Considering Hook law we can express $\delta\bar{U} = \delta U(\varepsilon_{ij})$. If forces $\bar{x}_i$ and $\bar{\sigma}_i^{(v)}$ are conservative (we suppose this), there exist potentials Φ and Ψ such that

- $\delta\Phi = \bar{x}_i\delta u_i$

- $\delta\Psi = \bar{\sigma}_i^{(v)}\delta u_i$.

By substituting these two relations to equation (4) we obtain expressions of virtual work:

$$\delta\left[\int_{V_\Omega}(\bar{U} + \Phi)dV + \int_{S_\Omega^\sigma}\Psi dS\right] = 0 . \qquad (5)$$

The expression in square brackets in (5) represents the whole potential energy Π. (5) represents stationarity condition of the functional

$$\Pi = \int_{V_\Omega}(\bar{U} + \Phi)dV + \int_{S_\Omega^\sigma}\Psi dS. \qquad (6)$$

In the following considerations we will focus to stationary task (constant volume and surface forces).

- $\Phi = \bar{x}_i u_i$

- $\Psi = \bar{\sigma}_i^{(v)}\delta u_i$.

By this, also the rearranged equation

$$\delta\Pi = \int_{V_\Omega}\bar{U}dV - \int_{V_\Omega}\bar{x}_i\delta u_i dV - \int_{S_\Omega^\sigma}\bar{\sigma}_i^{(v)}\delta u_i dS = 0$$

(7)

will stand as the stationarity condition of the functional Π. It can be also proved that $\delta(\delta\Pi) = \delta^2\Pi > 0$ which implies that in the equilibrium state the potential energy is minimal.

The dynamical formulation of the problem can be reached by using D'Alambert principle – by adding the inertia forces influence, so

$$\int_{V_\Omega} \sigma_{i,j} \delta\varepsilon_{ij} dV = \int_{V_\Omega} \bar{x}_i \delta u_i dV + \int_{S_\Omega^\sigma} \bar{\sigma}_i^{(v)} \delta u_i dS + \int_{V_\Omega} \rho u_{i,tt} \delta u_i dV \quad (8)$$

By integration by parts according to t between two time points t_1 and t_2 and supposing $\delta u_i(t_1) = 0$ we will obtain the equation

$$\int_{t_1}^{t_2} \delta(T - U) dt + \int_{t_1}^{t_2} \int_{V_\Omega} \bar{x}_i \delta u_i dV\, dt + \int_{t_1}^{t_2} \int_{S_\Omega^\sigma} \bar{\sigma}_i^{(v)} \delta u_i dS\, dt = 0 \quad (9)$$

where T is a kinetic energy, $T = \frac{1}{2} \int_{V_\Omega} \rho u_{i,t} . u_{i,t} dV$

and U is total deformation energy, $U = \int_{V_\Omega} \bar{U} dV$

4.2. Computer implementation

The solution of the task for the investigated domain, see fig.1, was done by using FEM software ANSYS, which provides solving various types of physical problems.

The problem was considered as axisymmetric, with y axis of symmetry = height in the notation of cylindrical coordinates. Radius goes along the x axis and the rotation goes perpendicularly to y axis, see fig.1.

Boundary conditions:

Symmetry along the rotation symmetry axis – axis y on the fig.1

Zero displacements in vertical direction of the points lying on horizontal plane of symmetry – UY=0 along x axis on fig.1

Load of 400 N on the upper area (represented by the upper line on the fig.2)

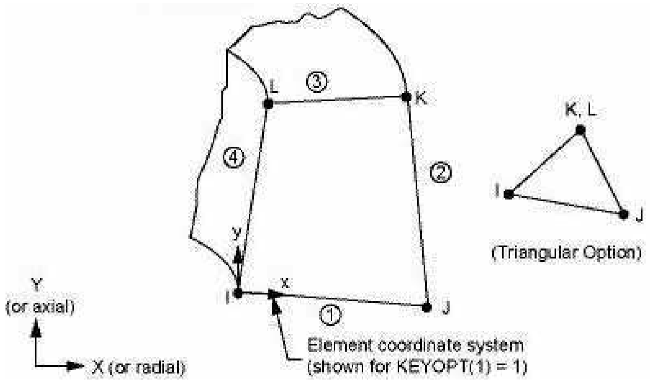


Fig.3. Element type used within FEM analysis.

Element type used *Plane25* – is used for 2-D modeling of axisymmetric structures with nonaxisymmetric loading. Examples of such loading are bending, shear, or torsion. The element is defined by four nodes having three degrees of freedom per node: translations in the nodal x, y, and z direction. For unrotated nodal coordinates, these directions correspond to the radial, axial, and tangential directions, respectively.

Material properties of four materials

	Cortical bone	Cancellous bone	Annulus fibrosis	Nucleus pulposis
Young's modul	1.2e9	1.5e8	1.e7	1.3e5
Poisson ratio	0.3	0.2	0.4	0.49999

4.3. The results

The pressure 600 N acting to the motion segment causes the deformation of the vertebra, see fig.3, maximal value of displacement are visible in the nodes representing tissue of annulus fibrosis on the line of symmetry (right bottom nodes of the domain in the picture). The maximal value is 0.146e-06.

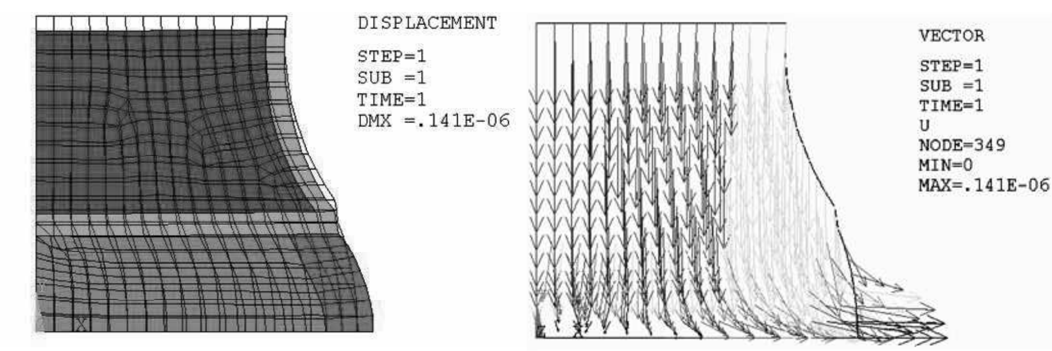
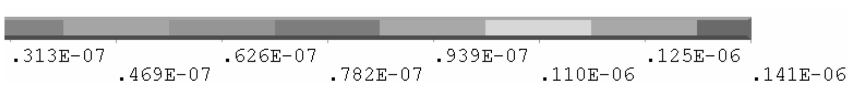


Figure 4: The displacement of the domain under specified load



5. CONCLUSIONS AND DISCUSSION

In this paper we addressed the semi stochastic approach to the 3d spine reconstruction and modeling in the equilibrium stage. It is clear that a further research would be of interest, e.g. in the directions of medical rehabilitation, or in the further theoretical considerations. In particular regularity conditions and regression related issues and exact confidence bounds for the parameters (based e.g. on method Potocký 1992) of a lumbar spine vertebras will be of interest.

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REFERENCES

[1] ALEMANY, P. A. AND ZANETTE, D. H. 1994. Fractal random walks from a variational formalism for Tsallis entropies. *Phys. Rev*. E 49 R956-R958.

[2] BROWN, T. et al. 1957. Some mechanical tests on the lumbosacral spine with particular reference to the intervertebral disc. In J. *Bone Int. Surg*., *39A*.

[3] DAMKOT, D.K. at al. 1984 The relationship between work history, work environment and low-back in men. In *Spine 9*, 395-399.

[4] ĎURIKOVIČ, V. AND ĎURIKOVIČOVÁ, M. 2005. Topological structure of solution sets to parabolic problems, *Topological Methods in Nonlinear Analysis, Vol. 25*, p. 313-349.

[5] EVANS, F.G. 1970. *Mechanical Properties of Bone*. First Edition, Thomas, Springfield, London (1973).

[6] FILUS, J., FILUS, L. AND STEHLIK, M. 2009. Pseudoexponential modelling of cancer diagnostic testing. *BIOMETRIE UND MEDIZINISCHE IN- FORMATIK GREIFSWALDER SEMINARBERICHTE* (ISBN 978-3-8322-7481-8 with SHAKER Publ.), Heft 15, 41-54.

[7] FRYMOYER, J.W. 1983. Risk factors in low back pain. In *Bone Joint Surg[Am] 65*, 213-218.

[8] FRYMOYER, J. W. et al: The Adult Spine. Principles and Practice. *Raven Press, Vol 2*, 1991

[9] GALANTE, J. et al. 1970. Physical properties of trabecular bone. Calc. Tissue Res., 5, 236-246.

[10] KELSEY, J.L. 1984. An epidemiological study of lifting and twisting on the job and risk for acute prolapsed lumbar intervertebral discs. In *Orthop. Res., 2*, 61-66.

[11] MARKOLF, K.L. 1971. Engineering characteristics of the human intervertebral joint. *PhD Thesis*. University of California.

[12] MARKOLF, K.L. 1972. K.L. Markolf. Deformation of the thoracolumbar intervertebral joints in response to external load. In j.*Bone Int. Surg., 54*, A.

[13] MCELHANEY, J. et al. A porous block model for cancellous bone, *ASME Publ. No 70*-WA/BHF-2.

[14] MÜLLER, WG. 2007. Collecting Spatial Data (3rd edn). *Physica-Verlag: Heidelberg*.

[15] NACHEMSON, A. 1960. Lumbar intradiscal pressure. In *Acta Orthop. Scand.* Suppl., 43.

[16] NACHEMSON, A. 1979. A.B. Schultz and M. Berkson. Mechanical properties of human lumbar spine motion segments, Part III: Influences of age, sex, disc level and degeneration. *Spine, 4* 1-8.

[17] POMERO, V., MITTON, D., LAPORTE, S., DE GUISE, J.A., SKALLI, W. 2004. Fast Accurate Stereoradiographic 3D-Reconstruction of the Spine Using a Combined Geometric and Statistic Model. *Clinical Biomechanics 19*(3): 240-247.

[18] POTOCKÝ, R.-VAN BAN, T. 1992. Confidence regions in nonlinear regression models. *Appl.Math. 37*, 29-39.

[19] ROLANDER, S.D. Motion of the lumbar spine with special reference to the stabilizing effect of posterior ROSS, S.M. 1995. Stochastic Processes. Wiley

[20] SOKOL, M. AND SUMEC, J. 2000. Simplified FEM Model of Human Spine and User Made Preprocesor for Input Data. In: *8th-AnSYS User Meeting*, Section C, 1-26 pages 1-6, Lednice (CZ). SVS FEM s.r.o., Brno.

[21] SONNERUP, L.A. 1972. A semi-experimental stress analysis of the human intervertebral disc in compression. *Exp. Mech 12*, 3, 142-147.

[22] STEHLÍK, M., RODRÍGUEZ-DÍAZ, J.M., MÜLLER, W.G. AND LÓPEZ-FIDALGO, J. 2008. Optimal allocation of bioassays in the case of parametrized covariance functions: an application in lung's retention of radioactive particles, *Test 17*: 56--68,

[23] STEHLÍK, M. 2009, Regularization of stereoradiographic 3D reconstruction of the spine, Stereology and Image Analysis. Ecs10 – In *Proceedings of the 10th European Congress of ISS,* (V.Capasso et al. Eds.), The MIRIAM Project Series, ESCULAPIO Pub. Co., Bologna, Italy, 2009, p.1-6

[24] STEHLÍK, M. 2007. Properties of stochastic representation of Poisson equation solution, *Journal of Applied Mathematics, Statistics and Informatics, Volume 2*, Number 2 (Dec. 2006), 149-153.

[25] STEHLÍK, M. 2009. Topological aggregation and dependence structures. *Ifas Res Rep*

[26] SUMEC, J., SOKOL, M. AND WENDLOVÁ, J. 1996. Biomechanics of Human Spine and Its Practical Using. In: *6th International Conference Biomechanics of Human '96*, pages 191-194, Sept. 17-19th.

[27] SUMEC, J. AND SOKOL, M. 1998. Mathematical Modelling of the human spine. In: *Engineering Mechanics '98*, pages 721-726, Svratka (CZ).

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