Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), **6** (2010), No. 1

ASYMPTOTIC STABILITY OF TIME VARYING DELAY-DIFFERENCE SYSTEM VIA MATRIX INEQUALITIES AND APPLICATION

KREANGKRI RATCHAGIT

Abstract

In this paper, we obtain some criteria for determining the asymptotic stability of the zero solution of time varying delay-difference system in terms of certain matrix inequalities by using a discrete version of the Lyapunov second method. The result has been applied to obtain new stability conditions for some classes of time varying delay-difference system such as delay-difference system of time varying delay-difference system with multiple delays in terms of certain matrix inequalities. Our results can be well suited for computational purposes.

Mathematics Subject Classification 2000: 39A11, 92B20
General Terms: Asymptotic Stability, Lyapunov Second Method, Time varying delay-difference system
Additional Key Words and Phrases: Asymptotic stability; Lyapunov function; Time varying delay-difference system; Matrix inequalities.

1. INTRODUCTION

An important class of difference systems is the class of nonlinear delay-difference system, where the delays are often appeared in the mathematical models of hereditary systems, Lotka-Volterra systems, control systems of growth of global economy, control of epidemics, etc. Thus models are usually described by a system of discrete-time system of the from

$$x(k+1) = f(x(k), x(k-h_1), x(k-h_2), \ldots, x(k-h_m)), \quad k = 0,1,2,\ldots,$$
$$x(k) = \phi(k), \quad k \in [-h_m, 0], \tag{1}$$

where $x(k) \in \mathbb{R}^n, 0 \le h_1 \le \ldots \le h_m, m \ge 1, \phi(k)$ is any given initial data, $f(x, x_1, \ldots, x_m) : \mathbb{R}^{n(p+1)} \to \mathbb{R}^n$
is a given n-vector function satisfying $f(0,0,\ldots,0) = 0$. This system is observed only at discrete times and the state at time $k+1$ is completely determined by the state at times $k, k-1, \ldots, k-h_p$.

The investigation of Lyapunov stability for difference systems has received considerable attention from many researchers. Many important results regarding this issue have been reported in the literature. One of most popular methods to study stability problem for system (1) is the one based on the discrete version of the Lyapunov second method [7, 8]. To the best of our knowledge, in most existing papers covering this problem the stability conditions were obtained based mainly on the estimation of the solution under suitable growth conditions. Thus at present, the problem of obtaining stability conditions via matrix inequalities for a more general class of delay-difference systems is of great theoretical and practical interest.

We consider time varying delay-difference system of the form

$$x(k+1) = A(k)x(k) + B(k)x(k-h), \tag{2}$$

where $x(k) \in \Omega \subseteq \mathbf{R}^n$, $h \geq 0$, $A(k)$ and $B(k)$ are the $n \times n$ matrices function.

The asymptotic stability of the zero solution of time varying delay-differential system has been developed during the past several years. We refer to monographs by Bay and Phat [2] and the references cited therein. Much less is known regarding the asymptotic stability of the zero solution of the time varying delay-difference system. Therefore, the purpose of this paper is to establish sufficient conditions for the asymptotic stability of the zero solution of (2) in terms of certain matrix inequalities.

2. PRELIMINARIES

The following notations will be used throughout the paper. $\mathbf{R}^+$ denotes the set of all non-negative real numbers; $\mathbf{Z}^+$ denotes the set of all non-negative integers; $\mathbf{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbf{R}^{n\times m}$ denotes the set of all $(n\times m)$-matrices; $M^{n\times m}$ denotes the space of all $(n\times m)$-matrices; and A^T denotes the transpose of the matrix A; A is the symmetric if $A = A^T$.

Matrix $Q \in {}^{n\times n}$ is positive semi definite $(Q \geq 0)$ if $x^T Q x \geq 0,$ for all $x \in \mathbf{R}^n$. If $x^T Q x > 0 (x^T Q x < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0, (Q < 0,$ resp.). It is easy to verify that $Q > 0,$ $(Q < 0,$ resp.) Iff

$$\exists \beta > 0: \; x^T Q x \geq \beta \|x\|^2, \forall x \in \mathbf{R}^n,$$

$$(\exists \beta > 0: \; x^T Q x \leq -\beta \|x\|^2, \forall x \in \mathbf{R}^n, \text{ resp.});$$

matrix function $Q(t) \in M^{n\times n}$ is positive definite if

$$\exists \beta > 0: \; x^T Q(t) x \geq \beta \|x\|^2, \; \forall t \in \mathbf{R}^+, \; x \in \mathbf{R}^n.$$

Fact 2.1 For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let us denote $V_\delta = \{x \in {}^n : \; \|x\| < \delta\}.$

Lemma 2.1 [9] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(k,x(k)):\mathbf{R}^n\to\mathbf{R}^+$ such that

$$\exists\beta>0:\Delta V(k,x(k))=V(k,x(k+1))-V(k,x(k))\le-\beta\|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k)\in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Lemma 2.2 [6] For any constant symmetric matrix $M\in {}^{n\times n}$, $M=M^T>0$, scalar $s\in {}^{+}/\{0\}$, vector function $W:[0,s]\to {}^{n}$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i))\ge\left(\sum_{i=0}^{s-1}w(i)\right)^T M\left(\sum_{i=0}^{s-1}w(i)\right).$$

We assume that the n-vector function nonlinear perturbations are bounded and satisfy the following hypotheses, respectively:

$$0\le\frac{f_i(r_1)-f_i(r_2)}{r_1-r_2}\le l_i,\quad \forall r_1,r_2\in\mathbf{R},\text{ and } r_1\ne r_2, \tag{2}$$

where $l_i>0$ are constants for $i=1,2,...,n$.

By assumption (2) we know that the functions $f_i(\cdot)$ satisfy

$$|f_i(x_i)|\le l_i|x_i|,\ i=1,2,...,n,$$

and

$$f^2{}_i(x_i)\le l_i x_i f_i(x_i),\quad i=1,2,...,n. \tag{3}$$

Fact 2.1 For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^Ty+y^Tx\le\varepsilon x^Tx+\varepsilon^{-1}y^Ty.$$

Lemma 2.1 [2] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x):\mathbf{R}^n\to\mathbf{R}^+$ such that

$$\exists\beta>0:\Delta V(x(k))=V(x(k+1))-V(x(k))\le-\beta\|x(k)\|^2,$$

along the solution of the system. In the case the above condition holds for all $x(k)\in V_\delta$, we say that the zero solution is locally asymptotically stable.

Lemma 2.2 [3] For any constant symmetric matrix $M\in\mathbf{R}^{n\times n}$, $M=M^T>0$, scalar $s\in\mathbf{Z}^+/\{0\}$, vector function $W:[0,s]\to\mathbf{R}^n$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i)) \geq \left(\sum_{i=0}^{s-1}w(i)\right)^T M\left(\sum_{i=0}^{s-1}w(i)\right).$$

3. MAIN RESULTS

In this section, we present the main results of this paper, which provides a sufficient condition for the asymptotic stability of the zero solution of (2) in terms of certain matrix inequalities.

Theorem 3.1 The zero solution of time varying delay-difference system (2) is asymptotically stable if there exist symmetric positive definite matrices function $P(k)$, $G(k)$, $W(k)$ and $L_1 = diag[l_{11},\ldots,l_{1n}] > 0,\ L_2 = diag[l_{21},\ldots,l_{2n}] > 0$ satisfying the following matrix inequalities:

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0, \qquad (3)$$

where

$$(1,1) = A^T(k)P(k+1)A(k) - P(k) + hG(k) + W(k) + \varepsilon A^T(k)P(k+1)P(k+1)A(k) + \varepsilon_1 A^T(k)A(k),$$

$$(2,2) = B^T(k)P(k+1)B(k) + (\varepsilon^{-1}+\varepsilon_2)B^T(k)B(k) - W(k),$$

$$(3,3) = -hG(k).$$

Proof. Consider the Lyapunov function

$$V(k,y(k)) = V_1(k,y(k)) + V_2(k,y(k)) + V_3(k,y(k)),$$

where

$$V_1(k,y(k)) = x^T(k)P(k)x(k),$$

$$V_2(k,y(k)) = \sum_{i=k-h+1}^{k}(h-k+i)x^T(i)G(k)x(i),$$

$$V_3(k,y(k)) = \sum_{i=k-h+1}^{k}x^T(i)W(k)x(i),$$

$P(k), G(k), W(k)$ being symmetric positive definite matrices function solutions of (3) and $y(k) = [x(k), x(k-h)]$.

Then difference of $V(k, y(k))$ along trajectory of solution of (2) is given by

$$\Delta V(k, y(k)) = \Delta V_1(k, y(k)) + \Delta V_2(k, y(k)) + \Delta V_3(k, y(k)),$$

where

$$\Delta V_1(k, y(k)) = V_1(k, x(k+1)) - V_1(k, x(k))$$

$$\begin{aligned}
&= [A(k)x(k) + B(k)x(k-h)]^T P(k+1)[A(k)x(k) + B(k)x(k-h)] \\
&\quad - x^T(k)P(k)x(k) \\
&= x^T(k)[A^T(k)P(k+1)A(k) - P(k)]x(k) \\
&\quad + x^T(k)A^T(k)P(k+1)B(k)x(k-h) + x^T(k-h)B^T(k)P(k+1)A(k)x(k) \\
&\quad + x^T(k-h)B^T(k)P(k+1)B(k)x(k-h),
\end{aligned}$$

$$\begin{aligned}
\Delta V_2(k, y(k)) &= \Delta\left(\sum_{i=k-h+1}^{k} (h-k+i)x^T(i)G(k)x(i) \right) \\
&= hx^T(k)G(k)x(k) - \sum_{i=k-h+1}^{k} x^T(i)G(k)x(i),
\end{aligned}$$

$$\begin{aligned}
\Delta V_3(k, y(k)) &= \Delta\left(\sum_{i=k-h+1}^{k} x^T(i)W(k)x(i) \right) \\
&= x^T(k)W(k)x(k) - x^T(k-h)W(k)x(k-h),
\end{aligned} \tag{4}$$

where **Fact 2.1** are utilized in (4), respectively.

Note that

$$\begin{aligned}
&x^T(k)A^T(k)P(k+1)B(k)x(k-h) + x^T(k-h)B^T(k)P(k+1)A(k)x(k) \le \\
&\quad \varepsilon x^T(k)A^T(k)P(k+1)P(k+1)A(k)x(k) + \varepsilon^{-1}x^T(k-h)B^T(k)B(k)x(k-h),
\end{aligned}$$

hence

$$\begin{aligned}
\Delta V_1(k, y(k)) \le\; & x^T(k)[A^T(k)P(k+1)A(k) - P(k) \\
& + \varepsilon A^T(k)P(k+1)P(k+1)A(k)]x(k) \\
& + x^T(k-h)[B^T(k)P(k+1)B(k) + \varepsilon^{-1}B^T(k)B(k)]x(k-h)
\end{aligned}$$

Then we have

$$\begin{aligned}\Delta V(k,y(k)) \le x^T(k)[A^T(k)P(k+1)A(k)-P(k)+hG(k)\\ +W(k)+\varepsilon A^T(k)P(k+1)P(k+1)A(k)]x(k)\\ +x^T(k-h)[B^T(k)P(k+1)B(k)+\varepsilon^{-1}B^T(k)B(k)\\ -W(k)]x(k-h)-\sum_{i=k-h+1}^{k} x^T(i)G(k)x(i).\end{aligned}$$

Using **Lemma 2.2.**, we obtain

$$\sum_{i=k-h+1}^{k} x^T(i)G(k)x(i) \ge \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG(k))\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right).$$

From the above inequality it follows that:

$$\begin{aligned}\Delta V &\le x^T(k)[A^T(k)P(k+1)A(k)-P+hG+W\\ &\quad +\varepsilon A^T(k)P(k+1)P(k+1)A(k)]x(k)\\ &\quad +x^T(k-h)[B^T(k)P(k+1)B(k)+\varepsilon^{-1}B^T(k)B(k)-W]x(k-h)\\ &\quad -\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG)\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)\end{aligned}$$

$$= \left(x^T(k), x^T(k-h), (\frac{1}{h}\sum_{i=k-h}^{k-1} x(i))^T \right)\begin{pmatrix}(1,1) & 0 & 0\\ 0 & (2,2) & 0\\ 0 & 0 & (3,3)\end{pmatrix}\begin{pmatrix}x(k)\\ x(k-h)\\ (\frac{1}{h}\sum_{i=k-h}^{k-1} x(i))\end{pmatrix}$$

$$= y^T(k)\psi y(k),$$

where

$(1,1)$=$A^T(k)P(k+1)A(k)-P+hG+W+\varepsilon A^T(k)P(k+1)P(k+1)A(k)$,
$(2,2)$=$B^T(k)P(k+1)B(k)+\varepsilon^{-1}B^T(k)B(k)-W(k)$,
$(3,3)=-hG(k)$,

,and $y(k)=\begin{pmatrix}x(k)\\ x(k-h)\\ \left(\frac{1}{h}\sum_{i=k-h}^{k-1} x(i)\right)\end{pmatrix}$.

By the condition (3), $\Delta V(k, y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1.** This completes the proof.

Example 3.1 Let us consider the time varying delay-difference system (2) of the form

$$x(k+1) = A(k)x(k) + B(k)x(k-h),$$

where the matrices are

$$A(k) = \begin{pmatrix} 1.9 - 0.5e^{-5.8t} - e^{-5.8t} & 1 \\ -e^{-5.8t} & 0.5e^{-t} - 2 \end{pmatrix}, B(k) = \begin{pmatrix} 3 - e^{6t} & -1 \\ 0 & -0.5e^{-2t} \end{pmatrix},$$

$\varepsilon = 0.5$, and $h = 1$.

Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 3.1** are feasible and

$$P(k) = \begin{pmatrix} e^{-5.8t} & 0 \\ 0 & 1 \end{pmatrix}, G(k) = \begin{pmatrix} 1 & 0 \\ 0 & e^{-0.5t} \end{pmatrix}, W(k) = \begin{pmatrix} e^{-t} & 0 \\ 0 & 1 \end{pmatrix}$$

are the set of solutions to the LMIs(5).

Therefore, the system is asymptotically stable.

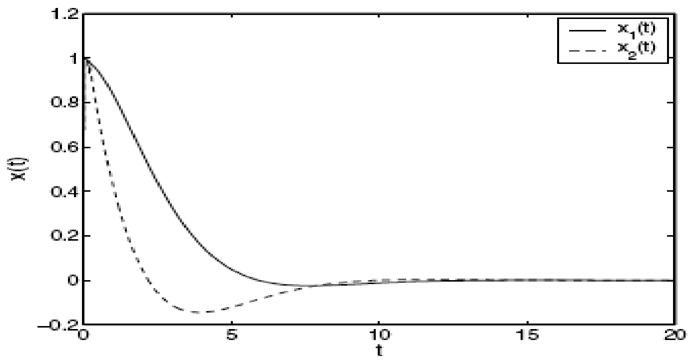

Fig. 1 Numerical simulation of a solution for the **example 3.1.**

For a given initial condition $x(\theta) = [1,1]^T$, convergence behavior of is shown in Fig. 1. As we can see from this figure, the steady state of nonlinear time varying delay-difference system is indeed asymptotically stable.

4. APPLICATION

In this section, we apply the main result of this paper, which provides a sufficient condition for the asymptotic stability of nonlinear time varying delay-difference system with multiple delays in terms of certain matrix inequalities.

We consider nonlinear time varying delay-difference system with multiple delays of the form

$$x(k+1)=A(k)x(k)+\sum_{i=1}^{m}B_i(k)x(k-h_i)+f(k,x(k),x(k-h_1),\ldots,x(k-h_m)), \tag{7}$$

where $x(k)\in\Omega\subseteq\mathbf{R}^n$, $0\le h_1\le\ldots\le h_m$, $m\ge 1$, $A(k)$ and $B_i(k)$, $i=1,2,...,m$ are the $n\times n$ matrices functions, $f(k,x(k),x(k-h_1),\ldots,x(k-h_m))$ is a n-vector function nonlinear perturbation satisfying $f(k,0,0,\ldots,0)=0$.

Theorem 4.1 The zero solution of the time varying delay-difference system (7) is asymptotically stable if there exist symmetric positive definite matrices $P(k),G_i(k)$, $W_i(k)$, $i=1,2,...,m$ and $L_i=diag[l_{i1},\ldots,l_{in}]>0$ satisfying the following matrix inequalities:

$$\psi=\begin{bmatrix}(0,0)&0&0&\cdots&0&0&0&0&\cdots&0\\0&(1,1)&(1,2)&\cdots&(1,m)&0&0&0&\cdots&0\\0&(2,1)&(2,2)&\cdots&(2,m)&0&0&0&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\0&(m,1)&(m,2)&\cdots&(m,m)&0&0&0&\cdots&0\\0&0&0&\cdots&0&(m+1,m+1)&0&0&\cdots&0\\0&0&0&0&0&0&(m+2,m+2)&0&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&0&\ddots&\ddots&\vdots\\0&0&0&0&0&0&\vdots&\ddots&\ddots&0\\0&0&0&0&0&0&0&\cdots&0&(2m,2m)\end{bmatrix}<0, \tag{8}$$

where

$$\begin{aligned}(0,0)=&A^T(k)P(k+1)A(k)-P(k)+\sum_{i=1}^{m}(h_iG_i(k)\\&+W_i(k))+\varepsilon A^T(k)P(k+1)P(k+1)A(k)+\varepsilon_1A^T(k)A(k)\\&+(\varepsilon_1^{-1}+\varepsilon_2^{-1}+1)L_0P(k+1)L_0,\\(1,1)=&B_1^T(k)P(k+1)B_1(k)+(\varepsilon^{-1}+\varepsilon_2)B_1^T(k)B_1(k)\\&+(\varepsilon_1^{-1}+\varepsilon_2^{-1}+1)L_1P(k+1)L_1-W_1(k),\\(1,2)=&B_1^T(k)P(k+1)B_2(k)+(\varepsilon^{-1}+\varepsilon_2)B_1^T(k)B_2(k),\end{aligned}$$

$(1,m) = B_1^T(k)P(k+1)B_m(k) + (\varepsilon^{-1}+\varepsilon_2)B_1^T(k)B_m(k)$,
$(2,1) = B_2^T(k)P(k+1)B_1(k) + (\varepsilon^{-1}+\varepsilon_2)B_2^T(k)B_1(k)$,
$(2,2) = B_2^T(k)P(k+1)B_2(k) + (\varepsilon^{-1}+\varepsilon_2)B_2^T(k)B_2(k)$
$\quad +(\varepsilon_1^{-1}+\varepsilon_2^{-1}+1)L_2P(k+1)L_2 - W_2(k),$
$(2,m) = B_2^T(k)P(k+1)B_m(k) + (\varepsilon^{-1}+\varepsilon_2)B_2^T(k)B_m(k)$,
$(m,1) = B_m^T(k)P(k+1)B_1(k) + (\varepsilon^{-1}+\varepsilon_2)B_m^T(k)B_1(k)$
$(m,2) = B_m^T(k)P(k+1)B_2(k) + (\varepsilon^{-1}+\varepsilon_2)B_m^T(k)B_2(k)$,
$(m,m) = B_m^T(k)P(k+1)B_m(k) + (\varepsilon^{-1}+\varepsilon_2)B_m^T(k)B_m(k)$
$\quad +(\varepsilon_1^{-1}+\varepsilon_2^{-1}+1)L_mP(k+1)L_m - W_m(k)$,
$(m+1,m+1) = -h_1G_1(k)$,
$(m+2,m+2) = -h_2G_2(k)$,
$(2m,2m) = -h_mG_m(k)$.

Proof Consider the Lyapunov function

$$V(k,y(k)) = V_1(k,y(k)) + V_2(k,y(k)) + V_3(k,y(k)),$$

where

$$V_1(k,y(k)) = x^T(k)P(k)x(k),$$
$$V_2(k,y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}(h-k+i)x^T(j)G_i(k)x(j),$$
$$V_3(k,y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}x^T(j)W_i(k)x(j),$$

$P(k), G_i(k)$ and $W_i(k)$, $i=1,2,...,m$ being symmetric positive definite solutions of (8) and $y(k) = [x(k), x(k-h_1),\ldots,x(k-h_m)]$.

Then difference of $V(k,y(k))$ along trajectory of solution of (7) is given by

$$\Delta V(k,y(k)) = \Delta V_1(k,y(k)) + \Delta V_2(k,y(k)) + \Delta V_3(k,y(k)),$$

where

$$\Delta V_1(k,y(k)) = V_1(k,x(k+1)) - V_1(k,x(k))$$

$$= [A(k)x(k) + \sum_{i=1}^{m} B_i(k)x(k-h_i) + f]^T P(k+1)[A(k)x(k) + \sum_{i=1}^{m} B_i(k)x(k-h_i) + f]$$
$$-x^T(k)P(k)x(k)$$
$$= x^T(k)[A^T(k)P(k+1)A(k) - P(k)]x(k)$$
$$+\sum_{i=1}^{m} x^T(k)A^T(k)P(k+1)B_i(k)x(k-h_i)$$
$$+\sum_{i=1}^{m} x^T(k-h_i)B_i^T(k)P(k+1)A(k)x(k)$$
$$+x^T(k)A^T(k)P(k+1)f + f^T P(k+1)A(k)x(k)$$
$$+\sum_{i=1}^{m} x^T(k-h_i)B_i^T(k)P(k+1)f + f^T P(k+1)B_i(k)x(k-h_i)$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)B_i^T(k)P(k+1)B_j(k)x(k-h_j) + f^T P(k+1)f ,$$

$$\Delta V_2(k, y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} (h_i - k + j)x^T(j)G_i(k)x(j)\right)$$
$$= \sum_{i=1}^{m} h_i x^T(k)G_i(k)x(k) - \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)G_i(k)x(j),$$

$$\Delta V_3(k, y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)W_i(k)x(j)\right)$$
$$= \sum_{i=1}^{m} x^T(k)W_i(k)x(k) - \sum_{i=1}^{m} x^T(k-h_i)W_i(k)x(k-h_i).$$

The rest of the proof is similar to that of **Theorem 3.1** need hold.

5. CONCLUSIONS

In this paper, based on a discrete analog of the Lyapunov second method, we have established a sufficient condition for the asymptotic stability of nonlinear time varying delay-difference system in terms of certain matrix inequalities. The result has been applied to obtain new stability conditions for some class of nonlinear time varying delay-difference system such as nonlinear time varying delay-difference system with multiple delays in terms of certain matrix inequalities.

ACKNOWLEDGMENTS

This work was partially done by the author during his research at the Hannover University (Germany), and the work is supported by the National Basic Program in Natural Sciences, Germany. The authors would like to thank anonymous referees for valuable comments, which improved the paper.

REFERENCES

[1] Agarwal, RP. (1992), Difference Equations and Inequalities, Marcel Dekker Inc., New York.

[2] Arik, S. (2005), Global Robust Stability Analysis of the Neural Networks with Discrete Time Delays, Chaos, Solutions & Fractals, Vol. 26, No. 5, pp. 1407-1414.

[3] Burton, TA. (1993), Averaged Neural Networks, Neural Networks, Vol. 6, pp. 677-680.

[4] Callier, F., Desoer, CA. (1992), Linear System Theory, Hong Kong, Springer-Verlag.

[5] Chua, LO., Yang, L. (1988), Cellular Neural Networks, IEEE Trans Circuits System, Vol. 35, pp. 1273-1290.

[6] Elaydi, S., Peterson, A. (1990), Difference Equations, Academic Press, New York.

[7] Gu, K. (2000), An Integral Inequality in the Stability Problem of Time-Delay Systems, In: Proc39th IEEE CDC Conf., Sydney, Australia, pp. 2805-2810.

[8] Gubta, MM., Jin, L. (1996), Globally Asymptotical Stability of Discrete-Time Analog Neural Networks, IEEE Trans Neural Networks, Vol. 7, No. 4, pp. 1024-1031.

[9] Hale, J. (1977), Theory of The Functional Differential Equations, Springer-Verlag, New York.

[10] Hu, S., Wang, J. (2006), Global Stability of a Class of Discrete-Time Recurrent Neural Networks, IEEE Trans Circuits and Systems-I, Vol. 49, No. 8, pp. 1021-1029.

[11] Infante, EF. (1981), A Lyapunov Functional for the Matrix Difference-Differential Equation, Journal of difference Equations, Vol. 35, No. 2, pp. 261-264.

[12] Liu, XZ., Liu, Y., Teo, K L. (2003), Stability Analysis of Impulsive Control Systems, Compute Math Model, Vol. 37, pp. 1357-1370.

[13] Ratchagit, K. (2007), Asymptotic Stability of Delay Difference System of the Hopfield Neural Networks via Matrix Inequalities and Application, International Journal of Neural Systems, Vol. 17, No. 5, pp. 425-430.

[14] Ratchagit, K. (2007), Asymptotic Stability of Delay Difference System of Cellular Neural Networks with Multiple Delays via Matrix Inequalities, KMITL Science Journal, Vol. 7, No. 2, pp. 18-27.

[15] Ratchagit, K. (2007), Asymptotic Stability of the Delay Difference System of the Cellular Neural Networks via Matrix Inequalities and Application, Advances in Computer Science and Engineering, Vol. 2, No. 1, pp. 51-59.

[16] Ratchagit, K. (2007), Asymptotic Stability of the Delay Difference System of the Cellular Neural Networks via Matrix Inequalities and Application, Advances in Differential Equations and Control Processes, Vol. 2, No. 2, pp. 171-177.

[17] Ratchagit, K. (2008), Asymptotic Stability of a Delay-Difference Control System of Hopfield Neural Networks via Matrix Inequalities and Applications, Walailak J Sci & Tech, Vol. 5, No. 2, pp. 191-202.

[18] Ratchagit, K. (2009), Asymptotic Stability of Delay-Difference System of Cellular Neural Networks with Time-Varying Delay via Matrix Inequalities, International Mathematical Forum, Vol. 4, No. 15, pp. 747-755.

[19] Ratchagit, K. (2009), Asymptotic Stability of Time Varying Delay-Difference System of Cellular Neural Networks with Time-Varying Delay via Matrix Inequalities and Applications, Int. J. of Appl. Math and Mech, Vol. 5, No. 3, pp. 11-21.

[20] Ratchagit, K. (2009), Asymptotic Stability of Delay-Difference Control System of Hopfield Neural Networks via Matrix Inequalities, International Mathematical Forum, Vol. 4, No. 28, pp. 1405-1411.

[21] Ratchagit, K. (2009), Asymptotic Stability of Time Varying Delay-Difference System via Matrix Inequalities, International Mathematical Forum, Vol. 4, No. 29, pp. 1443-1447.

[22] Ratchagit, K. (2009), Asymptotic Stability of Nonlinear Delay-Difference Control System with Time-Varying Delay via Matrix Inequalities, Applied Mathematical Sciences, Vol. 3, No. 30, pp. 1473-1478.

[23] Ratchagit, K. (2009), Asymptotic Stability of Delay-Difference Control System with Time-Varying Delay via Matrix Inequalities, Int. J. Contemp. Math. Sciences, Vol. 4, No. 20, pp. 999-1004.

[24] Wei, X P., Xu, J., Zang, Q. (2005), New Stability Conditions for Neural Networks with Constant and Variable Delays, Chaos, Solutions & Fractals, Vol. 26, No. 5, pp. 1391-1398.

[25] Ye, H. (1944), Global Stability and Local Stability of Hopfield Neural Networks with Delay, Physical Rev, Vol. 50, No. 5, pp. 4208-4215.

KREANGKRI RATCHAGIT
Department of Mathematics, Faculty of Science,
Maejo University,
Chiang Mai 50290, Thailand.

Received 2009