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BIAS CORRECTED BAYESIAN CONFIDENCE INTERVALS FOR THE EXPECTATION AND FOR THE VARIANCE OF LOGNORMAL DISTRIBUTIONS

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Abstract

Assuming X ~ LogN(μ,σ) and assuming a noninformative prior the posterior density of (μ,σ) is derived in order to get the dependent cumulative distribution functions of E(X) and of V(X). Then its α/2- and 1–α/2- quantiles are used to get confidence intervals for E(X) and for V(X) at a level of 1–α. Since its effective coverage probabilities differ from 1–α, a method is shown to adjust the biased cdf's. These now allow for constructing bias corrected confidence intervals.

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1. Introduction

If $\underline{x}$ is a sample from a LogN(μ,σ)- distribution, a traditional confidence interval CI covering the fixed value E(X) with probability 1–α, cannot be given. And so we use the cumulative distribution function (cdf) of the dependent random variable $Y = \exp[\mu + \sigma^2/2] \mid \underline{x}$ to get the α/2- and 1–α/2- quantiles for the limits of the Bayesian interval BI. This encloses the random variable $E(X)|\underline{x}$ with probability α* close to the theoretical α. Its difference is caused by a biased cdf and depends on the sample size and on σ. We present a method to reduce the bias of the cdf, which enables to construct a bias corrected BI^*. In addition a similar method is shown to get the cdf of the dependent random variable $Z = V(X)|\underline{x} = \exp[2\mu+\sigma^2]\cdot(\exp[\sigma^2] - 1) \mid \underline{x}$ and to construct a BI^* using the bias corrected cdf.

2. The posterior distribution of (μ,σ)

Following Bayes' theorem $p(\theta \mid x) \propto p(\theta)\cdot p(x \mid \theta)$
and using the fact of lnX ~ N(μ,σ))
we assume the noninformative prior $p(\mu,\sigma) \propto 1/\sigma$ for $\mu \in R$ and $\sigma \in R_+$
and multiply it by the the likelihood of $\underline{x}$ $L(\mu,\sigma \mid \underline{x}) \propto \prod_{i=1}^{n} \varphi(\ln x_i \mid \mu,\sigma)$.

Rewriting $L(\mu,\sigma \mid \underline{x}) \propto \frac{1}{\sigma^n} \cdot \exp\left[-\frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} \left((\ln x_i - \overline{x}_\ell) - (\mu - \overline{x}_\ell)\right)^2\right]$

and normalizing the product $p(\mu,\sigma)\cdot L(\mu,\sigma\,|\,\underline{x})$ yields the posterior density

$$f_{\mu\sigma}(\mu,\sigma\,|\,\underline{x}) = \frac{\sqrt{n}\cdot S_\ell^{n-1}}{\sqrt{\pi}\cdot 2^{(n-2)/2}\cdot \Gamma[(n-1)/2]\cdot \sigma^{n+1}}\cdot \exp\left[\frac{-1}{2\sigma^2}\left(S_\ell^2 + n(\mu-\overline{x}_\ell)^2\right)\right].$$

$\overline{x}_\ell$ denotes the mean of the logarithmised sample of size n. S_ℓ^2 denotes $(n-1)\cdot s_\ell^2$ and is n–1 times the variance of the logarithmised $\underline{x}$. It is obvious that $\overline{x}_\ell$ and s_ℓ^2 agree with the mean and variance of a sample from a N(μ,σ)- distribution.
For the sake of completeness the marginal posterior densities and some results based on it are studied: One-dimensional integrations of $f_{\mu\sigma}(\mu,\sigma\,|\,\underline{x})$ with respect to σ and to μ lead to the same marginal distributions of the transformed random variables as in the traditional case (without prior assumption): $\sqrt{n}(\overline{x}-\mu)/s \sim t_{n-1}$ and $(n-1)s^2/\sigma^2 \sim \chi^2_{n-1}$. This results in BI's agreeing with the CI's, and only its interpretations differ. The expected values agree as well:

$$E(\mu\,|\,\underline{x}) = \int_0^\infty\int_{-\infty}^\infty \mu\cdot f_{\mu\sigma}(\mu,\sigma)\,d\mu\,d\sigma = \overline{x}_\ell \quad \ldots \text{ for } n \ge 1,$$

$$E(\sigma^2\,|\,\underline{x}) = \int_0^\infty\int_{-\infty}^\infty \sigma^2\cdot f_{\mu\sigma}(\mu,\sigma)\,d\mu\,d\sigma = s_\ell^2\cdot\tfrac{n-1}{n-3} \quad \ldots \text{ for } n \ge 4.$$

Due to the independence of $(\overline{x}_\ell, s_\ell)$ the expectation of $\mu+\sigma^2/2\,|\,\underline{x}$ is $E(\mu\,|\,\underline{x}) + E(\sigma^2|\,\underline{x})/2$ and does exist, but the expectation of its antilog, which equals E(X)|$\underline{x}$, does not.

3. The posterior cdf's of Y = E(X) and of Z = V(X)

a) The dependent cdf of $Y = \exp[\mu+\sigma^2/2\,|\,\underline{x}\,]$ is calculated as the probability
$P(Y < y) = P(\mu+\sigma^2/2 < \ln y) = P(\mu < \ln y - \sigma^2/2 \wedge \sigma > 0) =$

$$F_y(y) = \int_0^\infty \int_{-\infty}^{\ln y-\sigma^2/2} f_{\mu\sigma}(\mu,\sigma\,|\,\underline{x})\,d\mu\,d\sigma$$

Substituting successively $\dfrac{\mu-\overline{x}_\ell}{\sigma/\sqrt{n}} = u$ and $\dfrac{S_\ell^2}{\sigma^2} = t$, the integral reduces to

$$F_y(y) = \int_0^\infty f(t;\chi^2_{n-1})\cdot\Phi\left(\sqrt{n}\cdot\left(\frac{\ln y-\overline{x}_\ell}{S_\ell}\cdot\sqrt{t} - \frac{S_\ell}{2\cdot\sqrt{t}}\right)\right)dt\,.$$

(3.1)
To obtain the posterior density $f_y(y)$, in the integrand above the cdf Φ has to be replaced by the standard normal pdf φ and to be multiplied by $\sqrt{n\cdot t}\,/(S_\ell\cdot y)$.

b) The dependent cdf of Z = $\exp[2\mu+\sigma^2]\cdot(\exp[\sigma^2]-1)\,|\,\underline{x}$ is calculated as the probability

$P(Z < z) = P(\,2\mu + \sigma^2 + \ln(\exp[\sigma^2]-1) < \ln z\,) = P(\,\mu < ul(\sigma) \wedge \sigma > 0\,)$, with the upper limit

$$ul(\sigma)=\left(\ln\frac{z}{\exp[\sigma^2]-1}-\sigma^2\right)/2\,.$$

The same substitutions as above reduce $F_z(z)=\int_0^{\infty}\int_{-\infty}^{ul(\sigma)} f_{\mu\sigma}(\mu,\sigma\,|\,\underline{x})\,d\mu\,d\sigma$ to

$$F_z(z)=\int_0^{\infty} f(t;\chi^2_{n-1})\cdot\Phi\left(\sqrt{n}\cdot\frac{ul(S_\ell/\sqrt{t})-\overline{x}_\ell}{S_\ell\cdot\sqrt{t}}\right)dt\,.$$

(3.2)

The argument of Φ as well can be written as $\dfrac{\sqrt{n}}{2}\left(\left(\ln\dfrac{z}{\exp[S_\ell^2/t]-1}-2\overline{x}_\ell\right)\cdot\dfrac{\sqrt{t}}{S_\ell}-\dfrac{S_\ell}{\sqrt{t}}\right)$.

To obtain the posterior density $f_z(z)$, in the integrand above the cdf Φ has to be replaced by φ and to be multiplied by $\sqrt{n\cdot t}\,/(2\cdot S_\ell\cdot z)$.

4. Bayesian confidence intervals for E(X) and for V(X)

The limits of the BI's at a level of $1-\alpha$ are calculated as $\alpha/2$- and $1-\alpha/2$- quantiles $F_y^{-1}(p)$ and $F_z^{-1}(p)$ respectively. The solution of the equation $F_y(y) = p$ with respect to y is found with an iterative method. To avoid a diverging algorithm a feasible starting point y_0 has to be found via a searching routine. It will be improved by Newton's method $y_{j+1} = y_j - (F_y(y_j) - p)\,/\,f_y(y_j)$, until the improvement is sufficiently close to zero.

Example 4.1

Assuming $\underline{x}$ = {1.5, 2, 2.5, 3.5, 5} is a sample from a LogN- distribution, we have ($\overline{x}_n$, s_n^2) = (2.9, 1.925) and ($\overline{x}_\ell$, s_ℓ^2) > (0.975, 0.222).

Choosing α = 0.9 and applying Newton's method yields the BI > (2.113, 6.279) for E(X), and the BI > (0.670, 75.153) for V(X). The two medians are $Y_{0.5}$ > 3.061 and $Z_{0.5}$ > 7.703. The following chapter helps to detect the cdf's $F_y(y)$ to be systematically underestimated, and the cdf's $F_z(z)$ to be slightly overestimated. This implies to shift the BI's to the left and to the right respectively. How to get the bias corrected BI's (2.065, 6.044) and (0.680, 78.034) respectively and the medians 2.974 and 2.786 will be shown detailed in chp. 6.

To get the shortest BI for E(X) at level $1-\alpha$ = 0.9 the same algorithm as in the case of a CI could be used. Since the distributions of Y and of Z surely are skewed right, the procedure starts at ul = $Y_{0.95}$ and has to lead to the left. Then the upper limit is to reduce stepwise until the difference ul – $F_y^{-1}\left(F_y(\mathrm{ul})-0.9\right)$ is minimal. For this example the shortest BI is (1.770, 5.050) for E(X) and is (0.234, 24.241) for V(X).

5. Detecting the biases of the cdf's

The main aim is to find the two effective error rates $\alpha^*/2$ of the BI's, and to construct a method approaching it close to $\alpha/2$. To get the effective error rates a great number N of samples fom a LogN(μ,σ)- distribution has to be drawn and each time the BI = (ll, ul) has to be calculated. Counting the cases of the lower limit ll exceeding E(X) (or V(X) respectively) and counting the cases of ul falling below it and dividing it by N would result in the onesided effective error rates. To avoid the time consuming calculations of the quantiles it is sufficient to consider only the empirical distribution of the cdf's. Since $F_t(T) \sim U(0,1)$ holds for any random variable T, and since $E(X)|\underline{x}$ and $V(X)|\underline{x}$ are regarded as random variables, the empirical cdf's $F_N(E(X))$ and $F_N(V(X))$ should be distributed standard uniformly.

If $F_y(E(X))$ is estimated correctly, its empirical cdf has to behave like a sample from a U(0,1)- distribution. If $F_y(E(X))$ is estimated biased, and if the differences $F_y(E(X))_{(j)}$ – j/N allow to be expressed by a function of (n,σ), this function will allow for adjusting the cdf's. The fact of the differences not to depend on μ is easily shown regarding the arguments ln(y) and ln(z) within the integrals above: asssuming $X_c \sim$ LogN(μ+c,σ) we have $E(X_c)$ = E(X)+c and the sample mean $\overline{x}_\ell + c$, and the constant cancels out.

To get an overview of the extent of the bias and of the difficulties to detect it, N = 40000 samples of size n = 5 from a LogN(μ,σ)- distribution were drawn. The parameters (μ,σ) are assumed to be $(\overline{x}_\ell, s_\ell)$ from example 4.1, and so we have E(X) > 2.967 and V(X) > 2.180. Choosing an arbitrary value of μ would not change the following outcomes.

The N values of $F_y(E(X))$ and of $F_z(V(X))$ are sorted and denoted by Fy and by Fz. Its empirical cdf's are to be compared with a straight line, which is the theoretical cdf. Graphing this line along with an empirical cdf hardly would uncover its differences, and by no means its characteristics could be found. And so we plot the differences $Fy_{(j)}$–j/N against $Fy_{(j)}$.

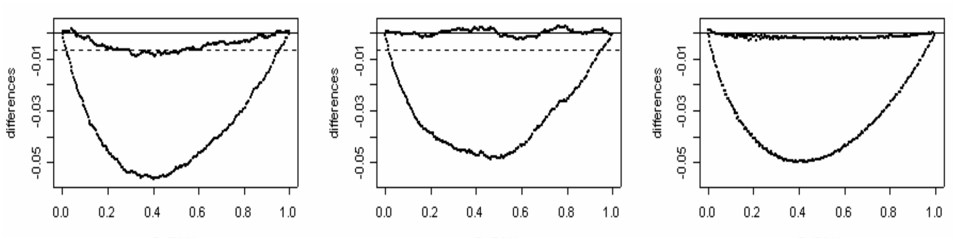

Fig. 5.1 Two examples of the differences $F_y(E(X))_{(j)}$ – j/N due to N = 40000 random samples of size n = 5.

Fig. 5.2 The differences due to 10000 systematic samples.

Instead of a solid line (a curve consting of N pieces) the figures above show the differences at 200 selected values $Fy_{(ind)}$ for ind = N·(i–½)/200 and i = 1,…200. These are the U- formed graphs. The dotted graphs at the top are the differences due to adjusted

cdf's and its derivations follow in the next chapter. The dashed lines drawn in label the 95%- quantile of the Kolmogorov-Smirnov statistic, testing $H_0 : Fy \sim U(0,1)$.
Figure 5.1 shows two examples of differences between the theoretical and empirical cdf of Y. Considering more examples shows varying graphs of differences and a sizeable sample would be nessecary to get things straight. To avoid this and using the known independence of $(\overline{x}_\ell, s_\ell)$, 100 p-quantiles of $\overline{x}_\ell \mid (\mu,\sigma)$ and of $s_\ell \mid \sigma$ are chosen to represent a systematic sample of size N = 100∗100. Choosing p = (i–½)/100 and i = 1,…,100 the quantiles are $\overline{x}_p = \mu + \Phi^{-1}(p)\cdot\sigma/\sqrt{n}$ and $s_p = \sqrt{F^{-1}(p;\chi^2_{n-1})}\cdot\sigma/\sqrt{n-1}$. The U- formed graph in fig. 5.2 and the э- formed graph in fig. 5.4 now show the differences using such a sample.

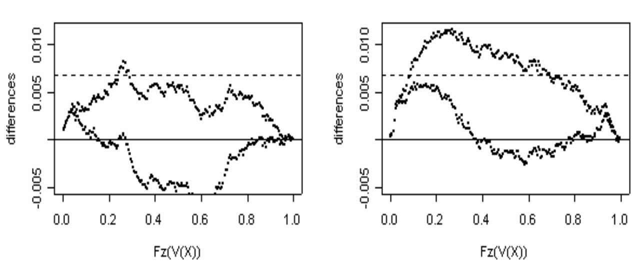

Fig. 5.3 Two examples of the differences $F_z(V(X))_{(j)} - j/N$ due to N = 40000 random samples of size n = 5.

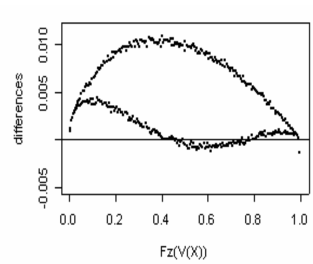

Fig. 5.4 The differences due to 10000 systematic samples.

6. Reducing the biases of the cdf's

Choosing any pair (n,σ) and using systematic samples, the graphs of the differences each time turn up to be unimodal. Now the differences can be approximated by $A\cdot F^{po}\cdot(1-F)^B$, in which F denotes $F_y(E(X))$ or $F_z(V(X))$ and A and B are functions of (n,σ). The power po was found to be nearly independent of (n,σ) and was fixed by po = 0.7.
To determine the factor A and the power B it is sufficient to know the mode md(n,σ) and the maximum value mv(n,σ) of the differences. Once md and mv are found it is easy to get $B = po\cdot(1/md - 1)$ and $A = mv/md^{po}/(1 - md)^B$. Once A and B are found the cdf's are adjusted by $F^* = F - A\cdot F^{po}\cdot(1-F)^B$ and allow for constructing bias corrected BI's. Using $md(n, s_\ell)$ and $mv(n, s_\ell)$, the dotted graphs near to zero of the six figures above show the reduced differences $Fy^*_{(j)} - j/N$ and $Fz^*_{(j)} - j/N$.
To determine md(n,σ) and mv(n,σ) eight sample sizes $2 \le n \le 30$ and 19 scale parameters σ with $-4 \le \ln\sigma \le 2.5$ were selected. Using all pairs (n,σ) and analysing systematic samples resulted in 8·19·2 graphs of differences $Fy_{(j)} - j/N$ and $Fz_{(j)} - j/N$. Its modes and maximum values were saved as four matrices of dimensions 8·19.

a) The modes of the differences $F_y(E(X))_{(j)} - j/N$ and of $F_z(V(X))_{(j)} - j/N$

Analysing the two matrices consisting of modes leaded to the approximating functions $md_y(n,\sigma) = 0.5 - \min(\sigma,1)/(n+2)$ and $md_z(n,\sigma) = 0.5 - 1/(n+3)$.

b) The maximum values of the differences $F_y(E(X))_{(j)} - j/N$

The points in figure 6.1 show the (negative) maximum values in the cases of n = (2, 3, 5) (left) and of n = (10, 15, 30). The solid lines are its approximations and are functions of (n,σ). To find out the type of these functions the eight graphs of maximum values were concerned commonly and $mv(n,\sigma) = \alpha\cdot\exp[(\ln\sigma-\beta)/\gamma - e^{(\ln\sigma-\beta)/\gamma}]$ was found to be a suitable type.
To find out the coefficients (α, β, γ), which are functions of n, the eight rows of the matrix consisting of the maximum values had to be concerned separately. The solutions of eight times minimizing $\sum_{i=1}^{19}\left(mv(n,\ln\sigma_i) - \exp[(\ln\sigma_i - \beta)/\gamma - e^{(\ln\sigma_i-\beta)/\gamma}]\right)^2$ with respect to (α, β, γ) leaded to a matrix C of dimension 8∗3. Now three functions had to be found which fit each column of C depending on n. This resulted in $\alpha(n) = -0.3/\sqrt{n-1.32}$ and in $\beta(n) = 0.063 - 0.72/(n-0.72)$ and in $\gamma(n) = 0.9 + 0.34/(n-1)$, holding for $n \geq 2$ and σ 2.72.
Checking the accuracy of these approximations showed it to be underestimated in the case of lnσ 1. In this region $mv_1(n,\sigma) = \alpha\cdot e^{-\beta/\gamma}$ was found to be a suitable function. Analysing a matrix C like the previous one resulted in $\alpha_1(n) = -0.055/\sqrt{n}$ and in $\beta_1(n) = 1.684 + 1.36/n^{1.5}$ and in $\gamma_1(\sigma) = \ln\sigma - 1$, holding for $n \geq 2$ and σ 2.72.

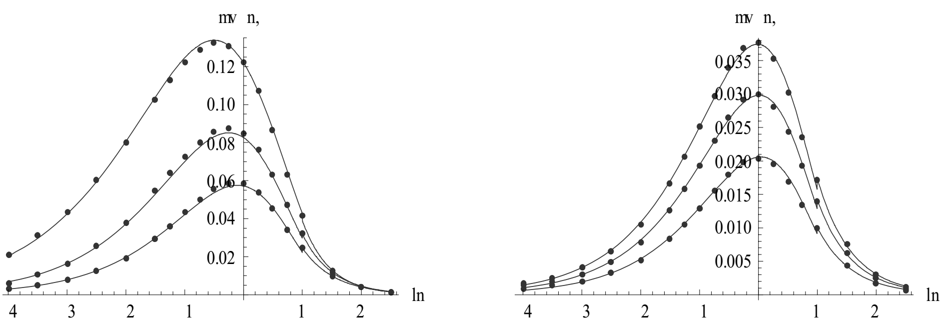

Fig. 6.1 The (negative) maximum values of the differences $F_y(E(X))_{(j)} - j/N$ in the cases of n = (2, 3, 5) (left) and n =(10, 15, 30).

Reconsidering example 4.1 and using $md(5,s_\ell)$ and $mv(5,s_\ell)$ to get A and B, and adjusting the cdf's by $Fy^* = Fy - A\cdot Fy^{po}\cdot(1-Fy)^B$, the three graphs close to zero in the figures 5.1 and 5.2 show the reduced differences. Using Fy^* to get the p-quantiles yields the bias corrected BI^* for E(X), already given in the example above.

c) The maximum values of the differences $F_z(V(X))_{(j)} - j/N$

The points in figure 6.2 show the extreme values in the cases of n = (2, 5, 15) (left) and of n = (3, 10, 30). The solid lines are its approximations and are functions of (n,σ). To find out the type of these functions the eight graphs of maximum values were concerned commonly and $mv_z(n,\sigma) = \alpha\cdot(\ln\sigma+\beta)\cdot\exp[-(\ln\sigma+\beta)^2]$ was found to be a suitable type.
To find out the coefficients (α, β), which are functions of n, the eight rows of the matrix consisting of the extreme values had to be concerned separately. Eight times estimating

(α, β) by the method of least squares leaded to a matrix C of dimension 8∗2. Now functions had to be found which fit each of the two columns of C depending on n. This resulted in $\alpha(n) = -0.067/\sqrt{n+1}$ and in $\beta = 0.3$, holding for $n \geq 2$ and $\sigma > 0$.

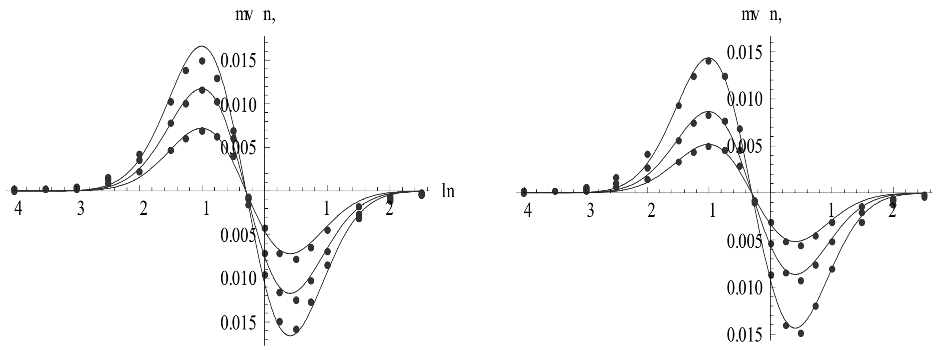

Fig. 6.2 The extreme values of the differences $F_z(V(X))_{(j)} - j/N$ in the cases of n = (2, 5, 15) (left) and n =(3, 10, 30).

Using $md_z(5, s_\ell)$ and $mv_z(5, s_\ell)$ to get A and B and adjusting the cdf's by $Fz^* = Fz - A \cdot Fz^{po.} \cdot (1-Fz)^B$, the three graphs close to zero in the figures 5.3 and 5.4 show the reduced differences. Using Fz^* to get the p-quantiles yields the bias corrected BI^* for V(X), already given in example 4.1.

7. The remaining bias

Concerning 24 selected pairs (n,σ), the cdf's of Y = E(X)|x and of Z = V(X)|x were calculated using (3.1) and (3.2) respectively. Adjusting Fy and Fz according to the rules given in chapter 5.b) and 5.c) significantly reduced the biases of Fy^* and of Fz^*, but these were not eliminated completely. The figures below show three out of 48 examples of remaining biases.

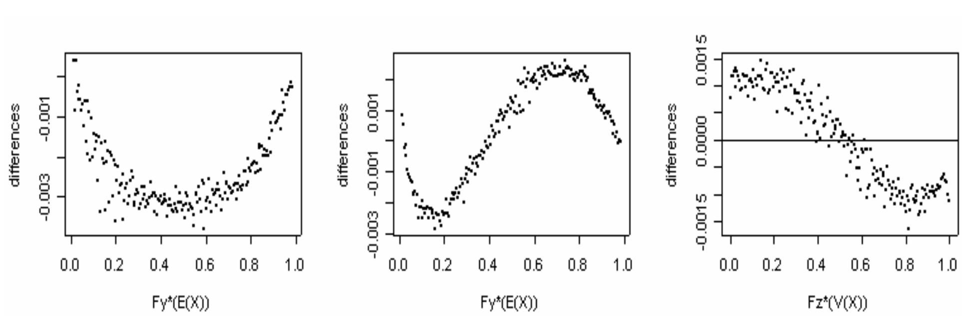

Fig. 7.1 The biases of the adjusted F_y^* (E(X)) in the cases of (n,σ) = (5, 0.61) and (10, 1.65 and (right) the bias of F_z^* (V(X)) in the case of (n,σ) = (20, 1).

Assuming a significance level of 1–α = 90% and considering the first two curves above at the value p = 0.05 shows $F_y^*(E(X))_{(p*N)} - p$ –0.001. The first bar of a histogram divided into 20 intervals would show a height of ca 5.1%. Solving $F_y^*(ll) = p$ with respect to the lower limit ll would lead to a BI^* with a onesided effective error of $\alpha^*/2$ 5.1%. Considering sample sizes n > 10, this error converges to α/2 = 5% and additionally depends on the shape parameter σ.
Instead of two times 24 graphs as above the following two tables show the remaining biases in terms of 4*6 pairs (b1, b2). The first value b1 denotes the maximal remaining bias $F_y^*(E(X))_{(j)} - j/N$, multiplied by 1000 and rounded. The second value denotes the mode(s) of the differences. If b1 is given together with two signs, this also implies two modes. And so (∓b1, b2) denotes a negative maximal bias of –b1/1000 at the mode b2, and a maximal positive bias of +b1/1000 at the mode 1–b2.

	–1.5	–1	–0.5	0	0.5	1	… lnσ
n	0.22	0.37	0.61	1	1.65	2.72	… σ
3	–3, 0.2	–6, 0.2	–9, 0.2	–14, 0.2	–15, 0.2	–10, 0.2	
5	+1, 0.4	–2, 0.4	–4, 0.5	–5, 0.2	–6, 0.2	∓½, 0.2	
10	+1, 0.2	+1, 0.2	–1, 0.5	–2, 0.3	∓3, 0.2	∓3, 0.2	
20	+1, 0.2	–1, 0.2	–1, 0.4	∓1, 0.2	∓2, 0.2	∓2, 0.2	

Table 7.1 The maximal remaining biases of $F_y^*(E(X))$
(given as one-tenths of a percent) and its modes.

For example we consider the case of n = 3 and σ = 1. Figure 6.1 shows the cdf's Fy to be afflicted with a maximal bias mv_y > –0.083 at the mode md_y > 0.3 (according to 5.a). The pair (–14, 0.2) of table 7.1 shows the maximal bias of Fy^* reduced to –0.014 at the (new) mode 0.2. Regarding histograms, the percental frequences of Fy < 0.3 would add up to 38.3%, and these of Fy^* < 0.2 would add up to 21.4%. The percental frequency h(Fy < 0.05) > 9.1% is reduced to h(Fy^* < 0.05) > 5.7%.
To explain the meaning of (∓3, 0.2) in the case of (n, σ) = (10, 1.65), at first we read from figure 6.1: $mv_y(10, 0.5)$ > –0.031. This maximum value (at the mode md > 0.42) is reduced to –0.003 and +0.003 respectively at the (new) modes 0.2 and 0.8. This is the abbreviated version of the second graph of figure 7.1. A confidence interval at level 1–α = 90% and based on Fy^* involves the onesided effective errors P(E(X) < ll) > 5.1% and P(E(X) > ul) > 5.05%.

The following table shows the remaining bias of $F_z^*(V(X))$:

	–1.5	–1	–0.5	0	0.5	1	… lnσ
n	0.22	0.37	0.61	1	1.65	2.72	… σ
3	∓4, 0.2	+4, 0.2	+8, 0.2	–8, 0.8	–6, 0.8	∓3, 0.2	
5	∓2, 0.2	+2, 0.2	+5, 0.2	–6, 0.8	–3, 0.8	∓2, 0.2	

10	$\mp$ 1.5, 0.2	+1, 0.2	±3, 0.2	±2, 0.2	–1, 0.5	$\mp$ 1, 0.2
20	$\mp$ 0.5, 0.2	–1, 0.6	±2, 0.2	±1.5, 0.2	–2, 0.5	$\mp$ 0.5, 0.2

Table 7.2 The maximal remaining biases of F_z^* (V(X)) *(given as one-tenths of a percent) and its modes.*

In the case of (n, σ) = (20, 1) the pair (±1.5, 0.2) corresponds to the third graph in figure 7.1. Compared to the maximal bias of Fz, which is ca. –0.01 (fig. 6.2), the maximal bias of Fz^* is reduced to +0.0015 at the mode 0.2 and to –0.0015 at the mode 0.8. A confidence interval at level 1–α = 90% and based on Fz^* involves two onesided effective errors of $\alpha^*/2$ 4.9%.

8. Summary

Methods are shown how to determine the posterior cdf's of E(X) and of V(X) and to construct Bayesian confidence intervals. The cdf's turn out to be biased and the figures 6.1 and 6.2 show the maximum values of the biases. Chapter 6 shows a method to reduce these biases and the tables 7.1 and 7.2 show the maximum values of the remaining biases. These tables provide to deduce approximately the onesided effective errors of the confidence intervals resulting from bias corrected cdf's. For sample sizes n 10, and additionally depending on the shape parameter σ, these errors are negligibly small.

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