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EXTENDED TANH METHOD & EXP-FUNCTION METHOD AND ITS APPLICATION TO (2 + 1)-DIMENSIONAL DISPERSIVE LONG WAVE NONLINEAR EQUATIONS

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Abstract

In this paper, Wu–Zhang equations (which describes (2 + 1)-dimensional dispersive long wave) equation is presented and the Extended Tanh Method and the Exp-Function Method are employed to the solution of nonlinear differential equations governing the problem. We demonstrate these methods provides powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. These methods can be used as an alternative to obtain analytic and approximate solution of different types of differential equations applied in engineering mathematics.

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Additional Key Words and Phrases: Extended Tanh Method; Exp-Function Method; Wu–Zhang equation; Non-linear equations

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1. INTRODUCTION

The nonlinear partial differential equations (NPDEs) are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry, biology, etc. Many efforts have been made on the study of NPDEs. Long wave in shallow water is a subject of broad interests and has a long colorful history. Physically, it has a rich variety of phenomenological manifestation, especially the existence of waves permanent in form and robust in maintaining their entities through mutual interaction and collision as well as the remarkable property of exhibiting recurrences of initial data when circumstances should prevail. These characteristics are due to the intimate interplay between the roles of nonlinearity and dispersion. Mathematically, it has been noted that validity of theoretical models critically depends on the domain of underlying key parameters characterize the specific motions to be modeled. In [1], Wu and Zhang derived three sets of model equations for modeling nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth. Their comparative study of these models is directed to explore the intrinsic properties in physical and mathematical terms that these models possess. Omitting the higher order terms, one of these equations, Wu–Zhang (WZ) equation, can be written as [1,2]

$$u_t + uu_x + vu_y + w_x = 0, \tag{1}$$

$$v_t + uv_x + vv_y + w_y = 0, \tag{2}$$

$$w_t + (uw)_x + (vw)_y + \tfrac{1}{3}(u_{xxx} + u_{xyy} + v_{xxy} + v_{yyy}) = 0, \tag{3}$$

where w is the elevation of the water, u is the surface velocity of water along x-direction, v is the surface velocity of water along y-direction. By scaling transformation and symmetry reduction, Eqs. (1–3).

A good understanding of Eqs. (1–3) is very helpful for coastal and civil engineers to apply the nonlinear water wave model in harbor and coastal design.

Recently introduced some new methods and are applied to equations such as Variation Iteration Method [3, 4], Homotopy Perturbation Method [5-11], Exp–Function method [12-17], the sine–cosine method [18-20], the homogeneous balance method [21], tanh–sech method [22-26] and extended tanh method [27-32].

In this paper, we intend to present implementation of extended tanh method to Wu–Zhang equation (2+1) -dimensional [33] equation that is solved by this method for the first time.

2. BASIC IDEA OF METHODS

2.1. Tanh and Extended Tanh Method

We now describe the tanh method for a given partial differential equation. This method was defined by Malfliet [22] and Fan and Hon [28]. Wazwaz summarized the main steps introduced for using this method as follows [26],

1- Wazwaz first considered a general form of nonlinear equation

$$N(u,u_t,u_x,u_{xx},u_{yy},u_{xxy},\ldots)=0, \tag{4}$$

2- To find the traveling wave solution of Eq. (4) he introduced thewave variable:

$$\xi = kx+\alpha y+\omega t, \tag{5}$$

so that:

$$u(x,y,t)=U(\xi), \tag{6}$$

And therefore, Eq. (4) constructs ODE of form

$$N(U,\omega U',kU',k^2U'',\alpha^2U'',k^2\alpha U''',\ldots)=0. \tag{7}$$

3- If all terms of the resulting ODE contain derivatives in ξ , then by integrating this equation, and by considering the constant of integration to be zero, we obtain a simplified ODE.

4- Introducing a new independent variable

$$Y=\tanh(\xi)\ (or\ Y=\coth(\xi)) \tag{8}$$

which leads to a change in the derivatives:

$$\begin{aligned} &\frac{d}{d\xi}=(1-Y^2)\frac{d}{dY},\\ &\frac{d^2}{d\xi^2}=(1-Y^2)\left(-2Y\frac{d}{dY}+(1-Y^2)\frac{d^2}{dY^2}\right),\\ &\frac{d^3}{d\xi^3}=(1-Y^2)\left((6Y^2-2)\frac{d}{dY}-6Y(1-Y^2)\frac{d^2}{dY^2}+(1-Y^2)^2\frac{d^3}{dY^3}\right), \end{aligned} \tag{9}$$

and the remaining derivatives may be derived similarly.

5- Introduce the ansatz and then solution of $U(\xi)$ is in the form of

Tanh Method:

$$U(\xi)=\sum_{p=0}^{m}a_pY^p=a_0+\ldots+a_mY^m \tag{10}$$

Extended Tanh Method:

$$U(\xi)=\sum_{p=-m}^{m}a_pY^p=a_{-m}Y^{-m}+...+a_0+...+a_mY^m \tag{11}$$

where m is positive integer which is unknown to be later determined, a_p is unknown constant.

6- To determine the parameter m, we usually balance linear terms of the highest order in the resulting equation with the highest order nonlinear terms. With m determined as described, equating the coefficients of powers of Y in the resulting equation will give a system of algebraic equations involving the a_p, $(p=-m,...,0,...,m)$, k,α and ω. Having determined these parameters, taking into account that in most cases these parameters are positive, we find, on using (11), an analytic solution in closed form.

The traveling wave solutions of many nonlinear ODEs and PDEs from soliton theory (and elsewhere) can be expressed as polynomials of hyperbolic or elliptic functions. For instance, the bell shaped sech-solutions and kink shaped tanh-solutions model wave phenomena in fluid dynamics, plasmas, elastic media, electrical circuits, optical fibers, chemical reactions, and bio-genetics.

2.2. The Rational Function in Exp-Function Method:

The Exp-Function Method was first proposed by He and Wu in 2006[12] and systematically studied in [34], and was successfully applied to KdV equation with variable coefficients [35], to high-dimensional nonlinear evolution equation [16], to burgers and combine KdV-mKdV (extended KdV) [36] equations, etc. In this section, we shall seek a rational function type of solution for a given partial equation, in terms of $\exp(\xi)$, of the following form:

$$U(\xi)=\sum_{p=0}^{m}\frac{a_p}{(1+e^{\xi})^p},\quad V(\xi)=\sum_{p=0}^{n}\frac{b_p}{(1+e^{\xi})^p},\quad W(\xi)=\sum_{p=0}^{l}\frac{c_p}{(1+e^{\xi})^p} \tag{12}$$

where a_p,b_p,c_p are some constants to be determined from the solution of Eqs. (1-3). Differentiating (12) with respect to ξ, introducing the result into Eqs. (1-3), and setting the coefficients of the same power of e^{ξ} equal to zero, we obtain algebraic equations.

3. EXACT SOLUTIONS OF WZ EQUATIONS

3.1. Using Extended Tanh Method:

By introducing a complex variation ξ defined as Eq. (5), $U(\xi)=\sum_{p=-m}^{m}a_pY^p$, $V(\xi)=\sum_{p=-n}^{n}b_pY^p$, $W(\xi)=\sum_{p=-l}^{l}c_pY^p$, Eqs. (1-3) become ordinary differential equations in the form of

$$\omega U'+kU'U+\alpha U'V+kW'=0. \tag{13}$$

$$\omega V'+kV'U+\alpha V'V+\alpha W'=0. \tag{14}$$

$$\omega W'+k(W'U+WU')+\alpha(V'W+VW')+\frac{1}{3}(k^3U'''+k\alpha^2U'''+k^2\alpha V'''+\alpha^3V''')=0. \tag{15}$$

Balancing W' term with $U'U$, W' term with $U'V$ in the first equation and W' term with VV' in the second equation or U''' term with $W'U$ in the third equation leads to the following ansatz: (we take $m=1,\ n=1,\ l=2$)

$$U(\xi)=\sum_{p=-1}^{1} a_p Y^p = a_{-1}Y^{-1}+a_0+a_1Y^1 \tag{16}$$

$$V(\xi)=\sum_{p=-1}^{1} b_p Y^p = b_{-1}Y^{-1}+b_0+b_1Y^1 \tag{17}$$

$$W(\xi)=\sum_{p=-2}^{2} c_p Y^p = c_{-2}Y^{-2}+c_{-1}Y^{-1}+c_0+c_1Y^1+c_2Y^2 \tag{18}$$

where $Y=\tanh(\xi)$ or $Y=\coth(\xi)$.

Substituting Eqs. (16-18) into Eqs. (13-15) and equating the coefficients of the powers Y then we get the following algebraic relations:

$-2kc_{-2}-\alpha a_{-1}b_{-1}-ka_{-1}^2=0,$

$-\omega a_{-1}-ka_{-1}a_0-kc_{-1}-\alpha a_{-1}b_0=0,$

$\alpha a_{-1}b_{-1}+ka_{-1}^2+2kc_{-2}+\alpha a_1b_{-1}-\alpha a_{-1}b_1=0,$

$\alpha a_{-1}b_0+kc_{-1}+\alpha a_1b_0+kc_1+ka_1a_0+\omega a_1+\omega a_{-1}=0,$

$-2kc_2-\alpha a_1b_1-ka_1^2=0,$

$-\omega a_1-ka_1a_0-kc_1-\alpha a_1b_0=0,$

$\alpha a_1b_1+ka_1^2+2kc_2+\alpha a_{-1}b_1-\alpha a_1b_{-1}=0,$

$-2\alpha c_{-2}-ka_{-1}b_{-1}-\alpha b_{-1}^2=0,$

$-\alpha b_{-1}b_0-kb_{-1}a_0-\omega b_{-1}-\alpha c_{-1}=0,$

$ka_{-1}b_1+2\alpha c_{-2}-ka_1b_{-1}+\alpha b_{-1}^2+ka_{-1}b_{-1}=0,$

$\alpha c_{-1}+\alpha b_{-1}b_0+\alpha c_1+\alpha b_1b_0+\omega b_{-1}+kb_{-1}a_0+\omega b_1+kb_1a_0=0,$

$-2\alpha c_2-ka_1b_1-\alpha b_1^2=0,$

$-\alpha b_1b_0-kb_1a_0-\omega b_1-\alpha c_1=0,$

$ka_1b_{-1}+2\alpha c_2-ka_{-1}b_1+\alpha b_1^2+ka_1b_1=0,$

$-2\alpha^3b_{-1}-2k^2\alpha b_{-1}-2k\alpha^2a_{-1}-3\alpha b_{-1}c_{-2}-3kc_{-2}a_{-1}-2k^3a_{-1}=0,$

$-\alpha b_0c_{-1}+3\alpha b_{-1}c_{-2}-\omega c_{-1}+\tfrac{8}{3}\alpha^3b_{-1}+3kc_{-2}a_{-1}-kc_0a_{-1}-kc_{-2}a_1+\tfrac{8}{3}k\alpha^2a_{-1}-kc_{-1}a_0$
$+\tfrac{8}{3}k^3a_{-1}-\alpha b_1c_{-2}+\tfrac{8}{3}k^2\alpha b_{-1}-\alpha b_{-1}c_0=0,$

$2kc_{-1}a_{-1}+2\alpha b_{-1}c_{-1}+2\alpha b_0c_{-2}+2\omega c_{-2}+2kc_{-2}a_0=0,$

$-2kc_{-2}a_0-2\alpha b_{-1}c_{-1}-2\omega c_{-2}-2kc_{-1}a_{-1}-2\alpha b_0c_{-2}=0,$

$-\tfrac{2}{3}\alpha^3b_1+\alpha b_1c_{-2}+\alpha b_0c_{-1}+\alpha b_{-1}c_2-\tfrac{2}{3}\alpha^3b_{-1}+kc_0a_1-\tfrac{2}{3}k^3a_1+kc_1a_0+$
$kc_{-1}a_0+\alpha b_{-1}c_0-\tfrac{2}{3}k^3a_{-1}+kc_0a_{-1}+\alpha b_1c_0-\tfrac{2}{3}k^2\alpha b_{-1}+\omega c_1-\tfrac{2}{3}k\alpha^2a_{-1}+$
$\alpha b_0c_1+\omega c_{-1}+kc_{-2}a_1-\tfrac{2}{3}k\alpha^2a_1+kc_2a_{-1}-\tfrac{2}{3}k^2\alpha b_1=0,$

$$-2\alpha^3 b_1 - 2k^2\alpha b_1 - 2k\alpha^2 a_1 - 3\alpha b_1 c_2 - 3kc_2 a_1 - 2k^3 a_1 = 0,$$
$$-2kc_2 a_0 - 2\alpha b_1 c_1 - 2\omega c_2 - 2kc_1 a_1 - 2\alpha b_0 c_2 = 0,$$
$$-\alpha b_0 c_1 + 3\alpha b_1 c_2 - \omega c_1 + {}^{8}\!/_{3}\,\alpha^3 b_1 + 3kc_2 a_1 - kc_0 a_1 - kc_2 a_{-1} + {}^{8}\!/_{3}\,k\alpha^2 a_1 - kc_1 a_0 \quad (19)$$
$$+ {}^{8}\!/_{3}\,k^3 a_1 - \alpha b_{-1} c_2 + {}^{8}\!/_{3}\,k^2 \alpha b_1 - \alpha b_1 c_0 = 0,$$
$$2kc_1 a_1 + 2\alpha b_1 c_1 + 2\alpha b_0 c_2 + 2\omega c_2 + 2kc_2 a_0 = 0,$$

The above equations are easy, but cumbersome to solve. Use of a modern computer algebra system may help and using, say Maple, gives:

Case1:

$$c_1 = 0,\ a_1 = \pm\frac{2\sqrt{3}}{3}k,\ c_{-1} = 0,\ c_{-2} = -\frac{2}{3}(k^2+\alpha^2),\ c_2 = -\frac{2}{3}(k^2+\alpha^2),$$
$$b_1 = \pm\frac{2}{3}\sqrt{3}\alpha,\ k = k,\ \omega = \omega,\ b_{-1} = \pm\frac{2}{3}\sqrt{3}\alpha,\ \alpha = \alpha,\ c_0 = \frac{4}{3}(k^2+\alpha^2), \quad (20)$$
$$a_{-1} = \pm\frac{2\sqrt{3}}{3}k,\ a_0 = -\frac{\omega+\alpha b_0}{k},\ \ b_0 = b_0$$

Substituting Eq. (20) into Eqs. (16-18), we get kinks solutions for Eq. (1-3) of the form

$$u_1(x,y,t) = -\frac{\omega+\alpha b_0}{k} \pm \frac{2\sqrt{3}}{3}k\tanh(\xi) \pm \frac{2\sqrt{3}}{3}k\coth(\xi) \quad (21)$$

$$v_1(x,y,t) = b_0 \pm \frac{2}{3}\sqrt{3}\alpha\tanh(\xi) \pm \frac{2}{3}\sqrt{3}\alpha\coth(\xi) \quad (22)$$

$$w_1(x,y,t) = \frac{4}{3}(k^2+\alpha^2) - \frac{2}{3}\left(k^2+\alpha^2\right)\tanh^2(\xi) - \frac{2}{3}\left(k^2+\alpha^2\right)\coth^2(\xi) \quad (23)$$

where $\xi = kx + \alpha y + \omega t$.

Case2:

$$c_1 = 0,\ a_1 = \mp\frac{2\sqrt{3}}{3}k,\ c_{-1} = 0,\ c_{-2} = -\frac{2}{3}(k^2+\alpha^2),\ c_2 = -\frac{2}{3}(k^2+\alpha^2),$$
$$b_1 = \mp\frac{2}{3}\sqrt{3}\alpha,\ \omega = \omega,\ b_{-1} = \pm\frac{2}{3}\sqrt{3}\alpha,\ \alpha = \alpha,\ c_0 = 0,\ a_{-1} = \pm\frac{2\sqrt{3}}{3}k \quad (24)$$
$$a_0 = -\frac{\omega+\alpha b_0}{k},\ b_0 = b_0,\ k = k$$

Substituting Eq. (24) into Eqs. (16-18), we get other kinks solutions for Eq. (1-3) of the form

$$u_2(x,y,t) = -\frac{\omega+\alpha b_0}{k} \mp \frac{2\sqrt{3}}{3}k\tanh(\xi) \pm \frac{2\sqrt{3}}{3}k\coth(\xi) \quad (25)$$

$$v_2(x,y,t) = b_0 \mp \frac{2}{3}\sqrt{3}\alpha\tanh(\xi) \pm \frac{2}{3}\sqrt{3}\alpha\coth(\xi) \quad (26)$$

$$w_2(x,y,t) = -\frac{2}{3}\left(k^2+\alpha^2\right)\left(\tanh^2(\xi) + \coth^2(\xi)\right) \quad (27)$$

where $\xi = kx + \alpha y + \omega t$. The properties of the solitary solutions u_2, v_2 and w_2 are shown in Fig. 1, 2 and 3.

Case3:

$$c_1=0,\ a_1=\pm\frac{2\sqrt{3}}{3}k,\ c_{-1}=0,\ b_1=\pm\frac{2\sqrt{3}}{3}\alpha,\ c_2=-\frac{2}{3}(k^2+\alpha^2),\omega=\omega,$$
$$b_{-1}=0,\ \alpha=\alpha,\ c_0=\frac{2}{3}(k^2+\alpha^2),\ a_{-1}=0,\ c_{-2}=0,\ a_0=-\frac{\omega+\alpha b_0}{k},\ b_0=b_0,\ k=k \tag{28}$$

Substituting Eq. (28) into Eqs. (16-18), we get other kinks solutions for Eq. (1-3) of the form

$$u_3(x,y,t)=-\frac{\omega+\alpha b_0}{k}\pm\frac{2\sqrt{3}}{3}k\tanh(\xi) \tag{29}$$

$$v_3(x,y,t)=b_0\pm\frac{2}{3}\sqrt{3}\alpha\tanh(\xi) \tag{30}$$

$$w_3(x,y,t)=\frac{2}{3}\left(k^2+\alpha^2\right)-\frac{2}{3}\left(k^2+\alpha^2\right)\tanh^2(\xi) \tag{31}$$

where $\xi = kx+\alpha y+\omega t$ or

$$u_4(x,y,t)=-\frac{\omega+\alpha b_0}{k}\pm\frac{2\sqrt{3}}{3}k\coth(\xi) \tag{32}$$

$$v_4(x,y,t)=b_0\pm\frac{2}{3}\sqrt{3}\alpha\coth(\xi) \tag{33}$$

$$w_4(x,y,t)={}^2\!/_3\left(k^2+\alpha^2\right)-{}^2\!/_3\left(k^2+\alpha^2\right)\coth^2(\xi) \tag{34}$$

where $\xi = kx+\alpha y+\omega t$.

If we choose $b_0=0, k=\sqrt{-R}, \omega=\mp\frac{2\sqrt{3}}{3}R, \alpha=0$, we have:

$$u_5(x,y,t)=\mp\frac{2\sqrt{3}}{3}\sqrt{-R}\pm\frac{2\sqrt{3}}{3}\sqrt{-R}\tanh(\xi) \tag{35}$$

$$v_5(x,y,t)=0 \tag{36}$$

$$w_5(x,y,t)=-\frac{2}{3}R+\frac{2}{3}R\tanh^2(\xi) \tag{37}$$

where $\xi=\sqrt{-R}(x\mp\frac{2\sqrt{3}}{3}\sqrt{-R}t)$ .The properties of the solitary solutions u_5, w_5 are shown in Fig.4, which are the exact solutions (1 + 1)-dimensional WZ equation [32].
In addition, when k, α, ω are imaginary numbers, the obtained solitary solution can be converted into periodic solution, so we define:

$$k=iK,\ \alpha=i\mathrm{A},\ \omega=i\Omega,\ i=\sqrt{-1} \tag{38}$$

Using the transformation:

$$\sinh(i\xi)=i\sin(\xi),\ \cosh(i\xi)=\cos(\xi),\ \tanh(i\xi)=i\tan(\xi),$$
$$\coth(i\xi)=-i\cot(\xi),\ \sec h(i\xi)=\sec(\xi),\ \csc h(i\xi)=-i\csc(\xi). \tag{39}$$

Then Eqs. (21-23,25-27,29-31,32-37) become:

$$u_6(x,y,t)=-\frac{\omega+\alpha b_0}{k}\mp\frac{2\sqrt{3}}{3}k\tan(\xi)\pm\frac{2\sqrt{3}}{3}k\cot(\xi) \tag{40}$$

$$v_6(x,y,t) = b_0 \mp \frac{2}{3}\sqrt{3}\alpha \tan(\xi) \pm \frac{2}{3}\sqrt{3}\alpha \cot(\xi) \quad (41)$$

$$w_6(x,y,t) = -\frac{4}{3}(k^2+\alpha^2) - \frac{2}{3}\left(k^2+\alpha^2\right)\tan^2(\xi) - \frac{2}{3}\left(k^2+\alpha^2\right)\cot^2(\xi) \quad (42)$$

where $\xi = kx + \alpha y + \omega t$.

$$u_7(x,y,t) = -\frac{\omega+\alpha b_0}{k} \pm \frac{2\sqrt{3}}{3}k\tan(\xi) \pm \frac{2\sqrt{3}}{3}k\cot(\xi) \quad (43)$$

$$v_7(x,y,t) = b_0 \pm \frac{2}{3}\sqrt{3}\alpha \tan(\xi) \pm \frac{2}{3}\sqrt{3}\alpha \cot(\xi) \quad (44)$$

$$w_7(x,y,t) = -\frac{2}{3}\left(k^2+\alpha^2\right)\left(\tan^2(\xi)+\cot^2(\xi)\right) \quad (45)$$

where $\xi = kx + \alpha y + \omega t$. The properties of the solitary solutions u_7, w_7 are shown in Fig. 5 and 6.

$$u_8(x,y,t) = -\frac{\omega+\alpha b_0}{k} \mp \frac{2\sqrt{3}}{3}k\tan(\xi) \quad (46)$$

$$v_8(x,y,t) = b_0 \mp \frac{2}{3}\sqrt{3}\alpha \tan(\xi) \quad (47)$$

$$w_8(x,y,t) = -\frac{2}{3}\left(k^2+\alpha^2\right) - \frac{2}{3}\left(k^2+\alpha^2\right)\tan^2(\xi) \quad (48)$$

where $\xi = kx + \alpha y + \omega t$.

$$u_9(x,y,t) = -\frac{\omega+\alpha b_0}{k} \pm \frac{2\sqrt{3}}{3}k\cot(\xi) \quad (49)$$

$$v_9(x,y,t) = b_0 \pm \frac{2}{3}\sqrt{3}\alpha \cot(\xi) \quad (50)$$

$$w_9(x,y,t) = -\frac{2}{3}\left(k^2+\alpha^2\right) - \frac{2}{3}\left(k^2+\alpha^2\right)\cot^2(\xi) \quad (51)$$

where $\xi = kx + \alpha y + \omega t$.

3.2. Using the Exp-Function Method:

Now we shall seek a rational function type of solution to the WZ equations, in terms of $\exp(\xi)$. By use of the exp-function method, after balancing we may choose the solution of Eq. (13-15) in the form:

$$U(\xi) = a_0 + \frac{a_1}{1+e^{\xi}} \quad (52)$$

$$V(\xi) = b_0 + \frac{b_1}{1+e^{\xi}} \quad (53)$$

$$W(\xi) = c_0 + \frac{c_1}{1+e^{\xi}} + \frac{c_2}{(1+e^{\xi})^2} \quad (54)$$

Differentiating Eq. (12) with respect to ξ , introducing the result into Eqs. (13-15), and setting the coefficients of the same power of e^{ξ} equal to zero, we obtain these algebraic equations:

$$-\omega a_1 - \alpha a_1 b_0 - k a_1 a_0 - \alpha a_1 b_1 - k a_1^2 - 2kc_2 - kc_1 = 0,$$
$$-\omega a_1 - k a_1 a_0 - kc_1 - \alpha a_1 b_0 = 0,$$
$$-\omega b_1 - \alpha b_1 b_0 - k b_1 a_0 - k a_1 b_1 - \alpha b_1^2 - 2\alpha c_2 - \alpha c_1 = 0,$$
$$-\omega b_1 - \alpha b_1 b_0 - k b_1 a_0 - \alpha c_1 = 0,$$
$$-\frac{1}{3}k\alpha^2 a_1 - k a_1 c_0 - \alpha b_1 c_0 - \frac{1}{3}k^2 \alpha b_1 - 3\alpha b_1 c_2 - \alpha b_0 c_1 - 2\alpha b_0 c_2 - 2k a_1 c_1$$
$$-2k a_0 c_2 - \frac{1}{3}k^3 a_1 - k a_0 c_1 - 2\omega c_2 - 2\alpha b_1 c_1 - 3k a_1 c_2 - \omega c_1 - \frac{1}{3}\alpha^3 b_1 = 0,$$
$$-2\alpha b_0 c_1 + \frac{4}{3}k\alpha^2 a_1 - 2k a_1 c_1 + \frac{4}{3}k^2 \alpha b_1 + \frac{4}{3}\alpha^3 b_1 - 2k a_0 c_1 - 2\omega c_1 - 2k a_1 c_0$$
$$-2k a_0 c_2 + \frac{4}{3}k^3 a_1 - 2\alpha b_0 c_2 - 2\omega c_2 - 2\alpha b_1 c_0 - 2\alpha b_1 c_1 = 0, \tag{55}$$
$$-\alpha b_0 c_1 - k a_0 c_1 - \frac{1}{3}k\alpha^2 a_1 - \frac{1}{3}k^3 a_1 - \omega c_1 - \frac{1}{3}k^2 \alpha b_1 - \frac{1}{3}\alpha^3 b_1 - k a_1 c_0 - \alpha b_1 c_0 = 0,$$

With the aid of Maple, the solutions of these algebraic equations are found to be:

$$a_0 = a_0,\ c_1 = \frac{1}{2}b_1^2 + \frac{2}{3}k^2,\ a_1 = \pm\frac{\sqrt{3}}{2}k,\ b_1 = b_1,\ c_2 = -\frac{1}{2}b_1^2 - \frac{2}{3}k^2,$$
$$\omega = \mp\frac{\sqrt{3}}{4}b_1^2 \mp \frac{\sqrt{3}}{3}k^2 \mp \frac{\sqrt{3}}{2}b_1 b_0 - k a_0,\ \alpha = \pm\frac{\sqrt{3}}{2}b_1,\ c_0 = 0,\ b_0 = b_0, k = k \tag{56}$$

Substituting Eq. (56) into Eq. (52-54), we obtain exact solutions for Eqs. (1-3) of the form:

$$u_{10}(x,y,t) = a_0 \pm \frac{\sqrt{3}k}{2(1+e^{\xi})} \tag{57}$$

$$v_{10}(x,y,t) = b_0 + \frac{b_1}{1+e^{\xi}} \tag{58}$$

$$w_{10}(x,y,t) = \left(\frac{1}{2}b_1^2 + \frac{2}{3}k^2\right)\left(\frac{1}{1+e^{\xi}} - \frac{1}{(1+e^{\xi})^2}\right) \tag{59}$$

where $\xi = kx \pm \frac{\sqrt{3}}{2}b_1 y + (\mp\frac{\sqrt{3}}{4}b_1^2 \mp \frac{\sqrt{3}}{3}k^2 \mp \frac{\sqrt{3}}{2}b_1 b_0 - k a_0)t$. Eqs.(57-59) will satisfy Eqs.(1-3).

4. CONCLUSION

In this paper, making use of a new more general ansatze, the Extended Tanh-Function Method is generalized, called generalized extended tanh-function method, for finding the exact solutions of NPDEs, and applied to (2+1)-dimensional Wu–Zhang eqation. The properties of the some soliton solutions for WZ equation are shown by some figures.The solution procedure is very simple, and the obtained solution is very concise. Its applications are worth further studying.

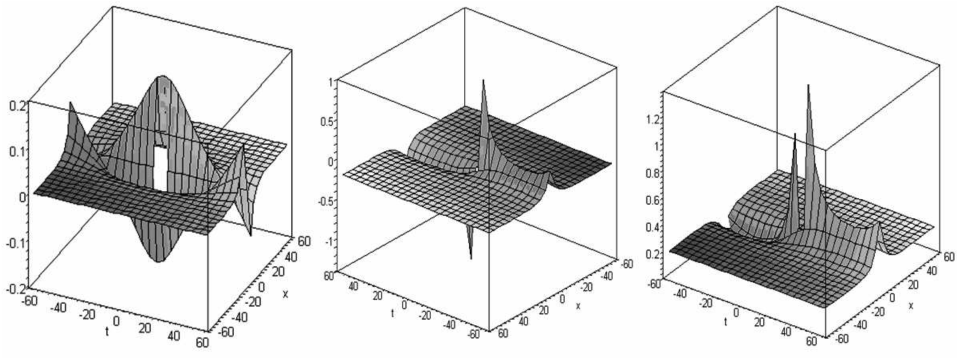

Fig. 1. The solitary solution u_2, the real part, imaginary part and the modulus, where $\alpha = b_0 = 0, k = 0.1, \omega = 0.02 i$.

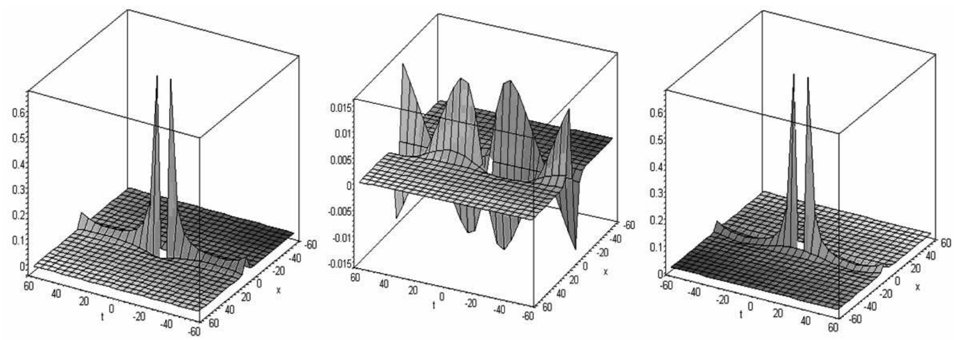

Fig. 2. The solitary solution w_2, the real part, imaginary part and the modulus, where $\alpha = b_0 = 0, k = 0.1, \omega = 0.02 i$.

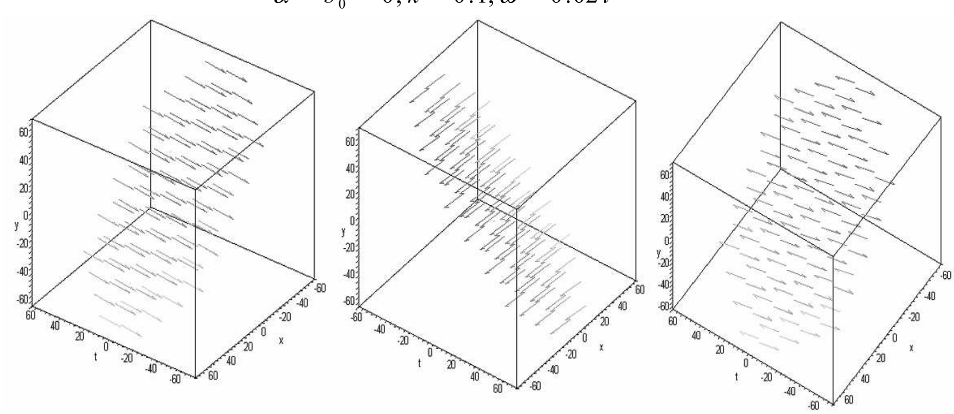

Fig. 3. The solitary solution u_2, v_2, w_2 where $\alpha = 0.2, b_0 = 0, k = 0.2, \omega = 0.2$.

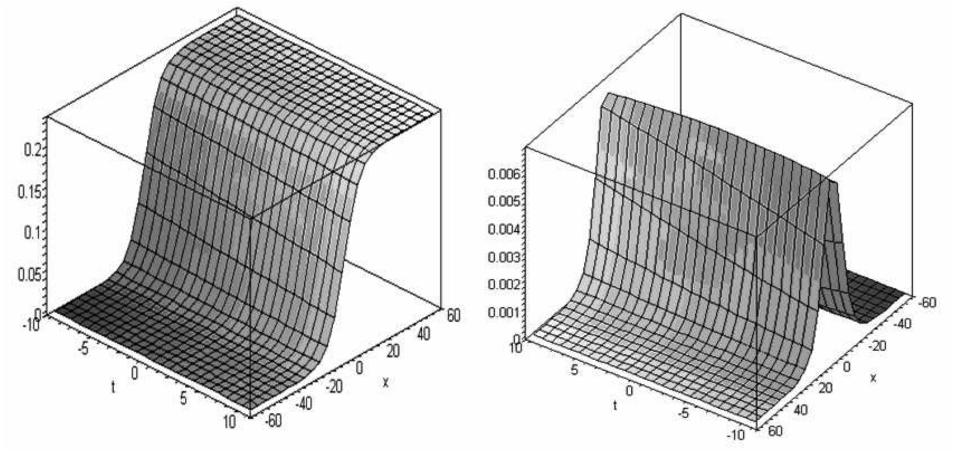

Fig. 4. The solitary solution u_5 and w_5, where $R = -0.01$.

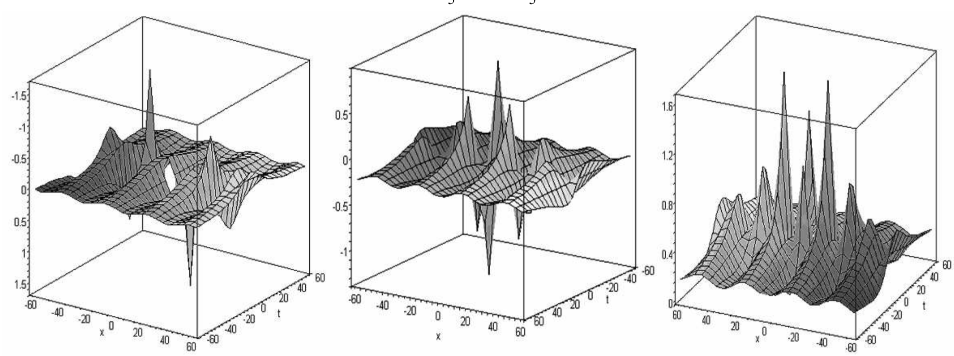

Fig. 5. The solitary solution u_7, the real part, imaginary part and the modulus, where

$$\alpha = b_0 = 0, k = 0.1, \omega = 0.02i.$$

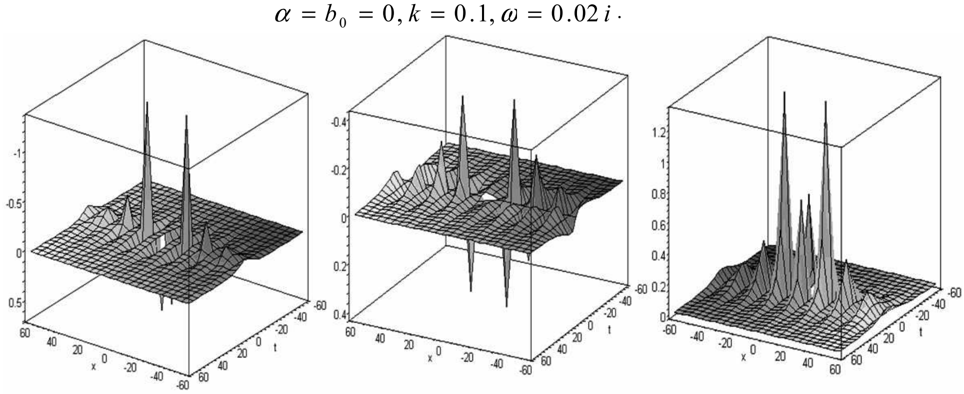

Fig. 6. The solitary solution w_7, the real part, imaginary part and the modulus, where

$$\alpha = b_0 = 0, k = 0.1, \omega = 0.02i.$$

REFERENCES

T.Y.Wu, J.E. Zhang, On modeling nonlinear long wave, in: L.P. Cook, V. Roytbhurd, M. Tulin (Eds.), Mathematics is for Solving Problems, SIAM, 1996, p. 233.

C.L. Chen, X.Y. Tang, S.Y. Lou, Exact solutions of (2+ 1)-dimensional dispersive long wave equation, in preparation.
J. H. He, Variational iteration method for autonomous ordinary differential systems, Applied Mathematics and Computation, 114 (2000) 115–123.
D.D. Ganji, H. Tari, H. Babazadeh, The application of He's variational iteration method to nonlinear equations arising in heat transfer, Physics Letters A, 363 (2007) 213–217.
J. H. He, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, International Journal of Non-Linear Mechanics, 35(2000) 37–43.
J. H. He, Application of homotopy perturbation method to nonlinear wave equations, Chaos Solitons Fractals, 26 (2005) 695–700.
J.H. He, Non-Perturbative method for strongly nonlinear problems. Berlin: dissertation. De-Verlag im Internet GmbH, 2006.
D.D. Ganji, The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer, Physics Letters A, A 355 (2006) 337–341.
D.D. Ganji, M. Rafei, Solitary wave solutions for a generalized Hirota–Satsuma coupled KdV equation by homotopy perturbation method, Physics Letters A, 356 (2006) 131–137.
D.D. Ganji, M.J. Hosseini and J. Shayegh, Some nonlinear heat transfer equations solved by three approximate methods. International communications in Heat and Mass Transfer, 34 (2007) 1003–1016.
D.D. Ganji, H. Tari, Approximate explicit solutions of nonlinear BBMB equations by He's methods and comparison with the exact solution, Physics Letters A, 367 (2007) 95–101.
Wu HX, J. H. He Exp-function method and its application to nonlinear equations, Chaos, Solitons & Fractals, 30 (3), (2006) 700–708.
Wu HX, J. H. He, Solitary solutions periodic solutions and compacton-like solutions using EXP-function method. Comput. Math. Wit xh App. 54 (7-8), (2007) 966–986.
J.H. He, Li-Na Zhang, Generalized solitary solution and compacton-like solution of the Jaulent–Miodek equations using the Exp-function method, Phys. Lett. A 372 (7), (2008) 1044–1047.
S. D. Zhu, Exp-function Method for the Discrete mKdV Lattice, International journal of Nonlinear Science and Numerical Solution, 8(3), (2007) 465–468.
S. Zhang, Application of Exp-function method to high-dimensional nonlinear evolution equation, Chaos, solution& fractals In Press (doi:10.1016/j.chaos.2006.11.014).
D.D. Ganji, A. Asgari, Z.Z. Ganji, Exp-Function Based Solution of Nonlinear Radhakrishnan, Kundu and Laskshmanan (RKL) Equation, Acta Appl Math. DOI 10.1007/s10440-008-9252-0.
A.M Wazwaz, The tanh and the sine–cosine methods for a reliable treatment of the modified equal width equation and its variants, Commun. Nonlinear Sci. Numer. Simul. 11 (2) (2006) 148–160.
A.M. Wazwaz, Solitary wave solutions for modified forms of Degasperis–Procesi and Camassa–Holm equations. Phys Lett A 2006;352(6):500–4.
A.M Wazwaz, The tanh method and the sine–cosine method for solving the KP-MEW equation, Int. J. Comput. Math. 82 (2) (2005) 235–246.
E. Fan, H. Zhang, A note on the homogeneous balance method, Phys. Lett. A 246 (1998) 403–406.
W. Malfliet, Solitary wave solutions of nonlinear wave equations, Amer. J. Phys. 60 (1992) 650–654.
W. Malfliet, W. Hereman, The tanh method: I. Exact solutions of nonlinear evolution and wave equations, Phys. Scr. 54 (1996) 569–575.
W. Malfliet, W. Hereman, The tanh method: II. Perturbation technique for conservative systems. Phys Scripta 1996;54:569–75.
A.M Wazwaz, The tanh method: solitons and periodic solutions for the Dodd–Bullough–Mikhailov and the Tziteica–Dodd–Bullough equations. Chaos, Solitons & Fractals 2005;25:55–63.
A.M Wazwaz, The tanh method: Exact solutions of the sine-Gordon and the sinh-Gordon equations. Appl Math Comput 2005;167(2):1196–210.
E Fan, Extented tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212.
E. Fan, Y.C. Hon, Generalized tanh method extented to special types of nonlinear equations, Z. Naturforsch 57a (2002) 692–700.
Xuedong Zheng, Yong Chen, Hongqing Zhang, Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation, Physics Letters A 311 (2003) 145–157.
A.M Wazwaz, The tanh method for travelling wave solutions to the Zhiber–Shabat equation and other related equations, Communications in Nonlinear Science and Numerical Simulation 13 (2008) 584–592.
M.A. Abdou, The extended tanh method and its applications for solving nonlinear physical models, Applied Mathematics and Computation 190 (2007) 988–996.
Xuedong Zheng, Yong Chen, Hongqing Zhang, Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation, Physics Letters A 311 (2003) 145–157.
Engui Fan, Uniformly constructing a series of explicit exact solutions to nonlinear equations in mathematical physics, Chaos Solitons and Fractals 16 (2003) 819–839.

J.H. He, M.A. Abdou, New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos Solitons Fractals 34 (5) (2007) 1421–1429.
S. Zhang, Application of Exp-function method to a KdV equation with variable coefficients, Phys. Lett. A 365 (5–6) (2007) 448–453.
A. Ebaid, Exact solitary wave solutions for some nonlinear evolution equations via Exp-function method, Phys. Lett. A 365 (3) (2007)213–219.
H. Demiray, A travelling wave solution to the Korteweg–de Vries–Burger equation, Appl. Math. Comput. 154 (2004) 665–670.

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