Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), **6** (2010), No. 1

A CONCEPT OF ADAPTIVE FUZZY REGRESSION MODEL VS. NEURO-RESPONSE SURFACE-BASED OPTIMIZATION FOR MATERIAL DISPATCHING IN A SINGLE STAGE NETWORK

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Abstract

This paper presents preliminary work done on simulation- based optimization of a stochastic material-dispatching sys-tem in a retailer network. The problem we consider is one of determining the optimal number of trucks and quantities to be dispatched in such a system. Theoretical solution models for versions of this problem can be found in the literature. Unlike most theoretical models, we can accommodate many real-life considerations, such as arbitrary distributions of the governing random variables, and all important cost elements, such as inventory-holding costs, stock-out costs, and transportation costs. Earlier the studies done by using two techniques, namely, neuro-response surfaces (NRSM) and The Mean Demand Heuristic (MDSM) [Subramaniam and Gosavi, 2004], for optimizing our system. We have also used a new model known as "Adaptive Fuzzy Linear Regression Model (AFLRM)" to provide us a benchmark for our other methods. Some computational results are also provided.

General Terms: Adaptive Fuzzy Regression, Neuro Response, Mean Demand Heuristic, Networks, Retail.
Mathematics Subject Classification 2000: 62J02

1. INTRODUCTION

Typically, a supply chain of consumer goods, such as gasoline, food products, and clothing items, consists of distribution centers, warehouses, and retailers. The distribution industry focuses on transporting goods from the manufacturer to the customers. A goal of this industry is to make the distribution process "lean", and thereby achieve cost benefits. The problem we have considered here is geared towards reducing the inventory in the distribution network and ensuring a satisfactory service level. Generally, a warehouse serves multiple retailers where customers arrive randomly to buy products. An optimization problem commonly-faced by managers is to determine (1) the number of trucks to be dispatched, and (2) the amount of goods to be dispatched.

Associated with this problem are the costs of holding excess inventory (inventory-holding costs), not being able to meet customer demand (stock-out costs), and transporting goods from the manufacturer to the retailers (transportation costs). The cost function that we have developed in this paper accounts for all of these elements. When there are multiple retailers and each retailer has unique random characteristics, such as arrival rate of customers and size of the demand, one has a large-scale and complex stochastic optimization problem on which it is not easy to construct an exact theoretical model.

Seminal work on this problem is from Clark and Scarf (1960). Jönsson and Silver (1987) have suggested the use of a "redistribution" strategy in which the inventories at the retailers are pooled and redistributed to standardize the inventory at each retailer.

McGavin, Schwarz, and Ward (1993) have suggested a so-called “between-replenishment," “risk-pooling" policy for this problem. Nahmias and Smith (1994) have developed a model for demands that have the negative-binomial distribution. Federgruen and Zipkin (1984) have modeled an extension of the previous work by Eppen and Schrage (1981) overcoming some of the limitations like normal distribution of the demand and identical holding and penalty costs across all the retailers. Kumar, Schwarz, and Ward (1995) have given static and dynamic models for a similar system but with a myopic allocation policy.

The last few years have seen an explosion in the number papers written on meta-heuristics and simulation-based optimization (Barton and Ivey 1996; Chen, Chen, and Yuce- san 2000; Fu and Hu 1997; Glassserman 1991; Glynn 2002; Ho and Cao 1991; Ho, Sreenivas and Vakili 1992; P ug1996; Shi and Ólafsson 1998; Spall 1992; Yan and Mukai 1992). Subramiam and Abhijit Gosavi have used two methods, namely, simulated annealing and neuro-response surfaces. They have also used an industrial heuristic, called the mean demand heuristic, which provides us with a benchmark for our methods and a starting point for simulated annealing. But the extension of the Subramaniam and Gosavi work is that we have developed another model “Adaptive Fuzzy Linear Regression Model (AFLRM)” which optimize the results in many ways.

A detailed description of the problem is given in Section 2. This is followed by the solution methodology in Section 3. Section 4 provides the computational results.

2. PROBLEM DESCRIPTION

The distribution network, generally, has a hierarchical structure in which a warehouse serves a set of retailers. All warehouses are coordinated and replenished by a central distribution center. We assume that distribution network has already been designed. The network can be divided into three echelons (see Figure 1). Echelon 1 is the first level from the manufacturer to the regional distribution centers, Echelon 2 is the level from the regional distribution centers to the local distribution centers, and Echelon 3 is the level from the local distribution centers to the retailers. Our focus in this paper is on a problem in Echelon 3. A “transshipment" point is a point in a supply chain where goods are transferred from one echelon to another. Goods are stored temporarily at the transshipment point. The warehouse acts as a transshipment point for the items to be distributed, and goods received by it are distributed among the retailers. The problem is to determine the optimal quantities to be delivered from the transshipment point to each of the retailers so as to minimize the average cost of operating the system. Our model takes the following costs into consideration. (1) Inventory-holding costs. (2) Stock- out costs, which include the cost of lost sales and the loss
of goodwill. (3) Transportation costs, which include the operating cost of the truck and the cost of transporting the goods, which in turn depends on the quantities transported.

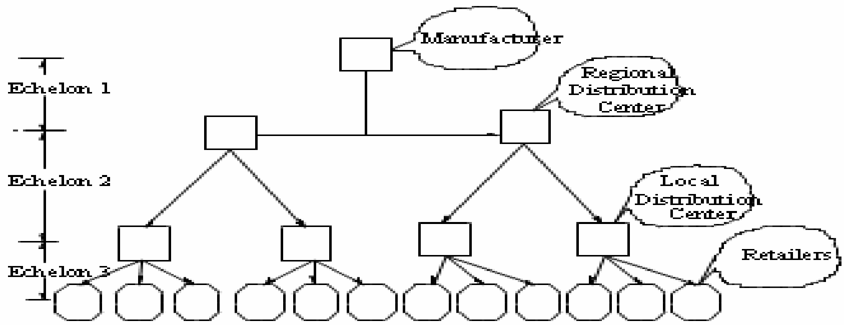


Figure 1: Distribution Network

The system considered here is a complex real-world system. The random variables governing our system are: the inter-arrival time of each customer, the quantity demanded by each customer, the service time for each truck, and the travel time between the warehouse and retailers and the same between the retailers. It is difficult to develop an exact mathematical model that will account for all the random variables and costs that we have considered. This motivates a simulation-based model for performance evaluation and optimization. We have assumed that each retailer is distinct and has unique values for the system parameters. The rate of arrival of customers, the inventory-holding costs, and the stock-out costs are all different for each retailer. Our objective is to minimize the average cost per unit time of running the entire system. We next present a mathematical description of the problem.

Consider a probability space (Ω, F, P), where P de- notes the distribution of the profits generated by the system under consideration. From the simulator of the profits generated by the system, one can generate k samples: *($\omega^1, \omega^2,..., \omega^k$). Let q = (q1,q2,...,qn)* denote the vector of delivery quantities, i.e.*, the delivery vector,* where qi denotes the quantity to be delivered to the ith retailer and n is the number of retailers. Let $f_i(\vec{q}, \omega^j)$ denote the average cost per unit time of running the system estimated from the jth sample (generated by the simulator) when the delivery vector is $\vec{q}$. Let C_t denote the truck operating cost per unit quantity per unit time. Let $L_i(\vec{q}, \vec{q}^{\,j})$ denote the total number of lost sales at the ith retailer in the jth sample when the delivery vector is $\vec{q}$, and C_l^i denote the lost-sales or stock-out cost per unit quantity at the ith retailer. Let $I_i(\vec{q}, t, \omega^j)$ denote the positive inventory at time t at the ith retailer in the jth sample, C_e^i denote the inventory-holding cost per unit quantity at the ith retailer, and $T(\omega^j)$ denote the length of the trip in the jth sample. Then mathematically, the problem is to:

Minimize $\quad C_t \sum_{i=1}^{n} E\left[f_i\left(\vec{q}, \omega^j\right)\right]$

where $\quad f_i\left(\vec{q}, \omega^j\right) = C_l L_i\left(\vec{q}, \omega^j\right) + \dfrac{C_e \displaystyle\int_0^{T(\omega^j)} I_i\left(\vec{q}, t, \omega^j\right) dt}{T(\omega^j)}$

and $\quad E\left[f_i\left(\vec{q}, \omega^j\right)\right] = \lim\limits_{k \to \infty} \sum_{j=1}^{k} f_i\left(\vec{q}, \omega^j\right)$

such that $q_i \geq 0$ for $i = 1, 2, \ldots\ldots\ldots\ldots, N$.

3. SOLUTION METHODOLOGIES

The two optimization techniques, namely, Neuro-Response Surface Methods and the Adaptive Fuzzy Linear Regression Model, which we have used to solve the above problem:

3.1. Neuro-Response Surfaces

The response surface method (RSM) has been a popular method of optimization in simulation-based optimization due to its robustness and strong mathematical (statistical) backing. Traditional RSM uses regression for fitting the objective function. This requires the assumption of a metamodel. Unlike traditional RSM, NRSM does *not* assume a metamodel. In NRSM, function fitting is done using neural networks. Its power lies in its ability to fit *any* surface. The NRSM uses the well-known backpropogation algorithm. The steps involved in this algorithm are explained below.

Consider a neural network shown in Figure2. The so-called "input" layer consists of a finite number of nodes; usually one node is associated with each decision variable. The number of nodes in the hidden layer is a function of the non-linearity of the function to be fitted. (Greater the non-linearity of the function larger is the required number of nodes in the hidden layer.) The neural network computes the so-called "weights" which represent its metamodel. Let $w(i, h)$ denote the weight from the ith input node to the hth hidden node and $x(h)$ the weight from the hth hidden node to the output node. The bias node is comparable to the constant term in regression-based function fitting. Let p denote the number of pieces of data used for training the neural net. The available data for the pth piece is $(\vec{u}_p, y_p)$ where $\vec{u}_p$ denotes a vector with I components.

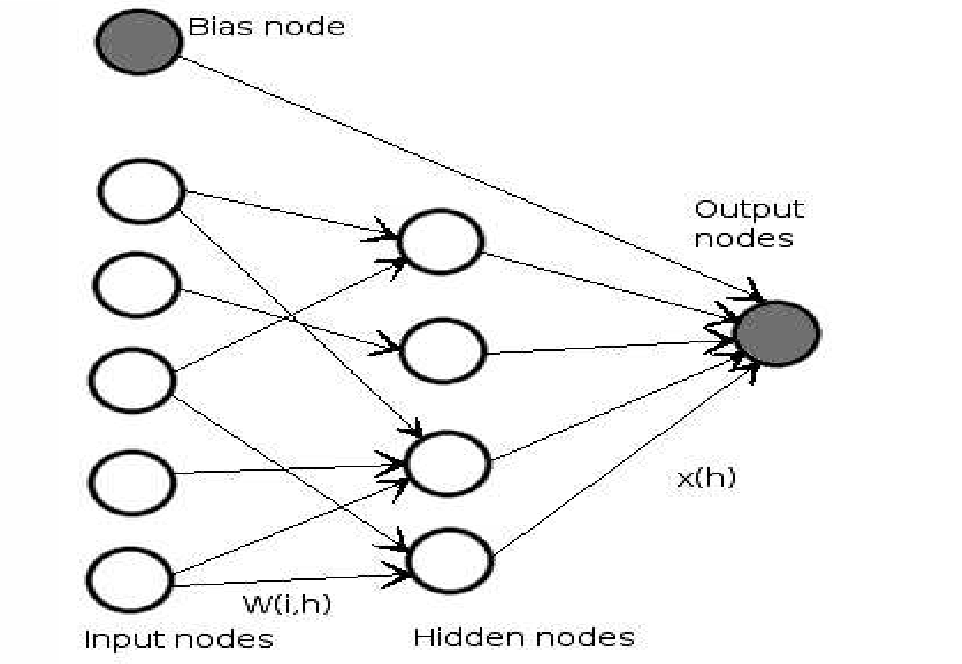


Figure: 2

Step 1: The first step is to set all the weights to small random numbers. Set the value of the SSE (sum of squared errors) to a large number.

Step 2: Calculate the output value at the hidden node as follows:

$$V_p^*(h)=\sum_{i=1}^{I} w(i,h)u_p(i)$$

where, $V_p^*(h)$ denotes the output value at the hidden node h and $u_p(i)$ the input value to the i_{th} input node.

Step 3: Compute $V_p(h)$ as follows, using the sigmoid function.

$$V_p(h)=\frac{1}{1+e^{-V_p^*(h)}}$$

Step 4: Compute each of the output terms Q_p, for p = 1,2,3, …..,n where n is the number of data pieces, using

$$O_p = b+\sum_{h=1}^{H} x(h)V_p(h).$$

Step 5: Update b, w(i, h), x(h) as follows

$$b \leftarrow b + \mu \sum_{p=1}^{n} (y_p - O_p).$$

$$w(i,h) \leftarrow w(i,h) + \mu \sum_{p=1}^{n} (y_p - O_p) x_h v_p(h)(1 - v_p(h)) u_p(i).$$

$$x(h) \leftarrow x(h) + \mu \sum_{p=1}^{n} (y_p - O_p) v_p(h).$$

Step 6: Calculate SSE_{new} using

SSE_{new} =
Reduce the value of μ. If $|SSE_{new} - SSE_{old}|$ < *tolerance*, Stop. Otherwise, set $SSE_{old} = SSE_{new}$ and return to Stp 2.

3.2. Underlying theories for the adaptive fuzzy logistic regression model:

3.1.1. Fuzzy linear regression theory

Regression analysis is an estimation method used in finding a crisp relationship between the dependent and independent variables and also used to estimate the variance of measurement error. Fuzzy regression analysis is an extension of the classical regression analysis in which some elements of the models are represented by fuzzy numbers [3]. There are two categories of fuzzy regression analysis; the first is a possibilistic regression analysis which is based on possibility concepts. Possibilistic regression analysis uses fuzzy linear system as a regression model whereby the total vagueness of the estimated values for the dependent variables is minimized. It was first proposed by Tanaka et al. [1, 3]. The second category of fuzzy regression analysis adopts the fuzzy least squares method (FLSM) for minimizing errors between the given outputs and the estimated outputs.

3.1.2. Statistical logistic regression theory

Logistic regression is a mathematical modeling approach that is used to describe the relationship between several explanatory variables X's to a dichotomous dependent variable Y [5]. Logistic regression can be used to predict the outcome from a set of variables that may be continuous, discrete, dichotomous, or a mix of any of these. That is, logistic regression makes no assumption about the distribution of the independent variables. They do not have to be normally distributed, linearly related or of equal variance within each group. The dichotomous dependent variable can take the value of 1 with a probability of success P, or the value of 0 with probability of failure 1-P. This type of variable is called Bernoulli (or binary) variable. The relationship between the predictor and response variables is not a linear function in logistic regression, instead, logistic regression function is used which is the logit transformation of P[5]

$$Ln(\frac{P}{1+P}) = a + b_1x_1 + b_2x_2 + ... + b_jx_j$$

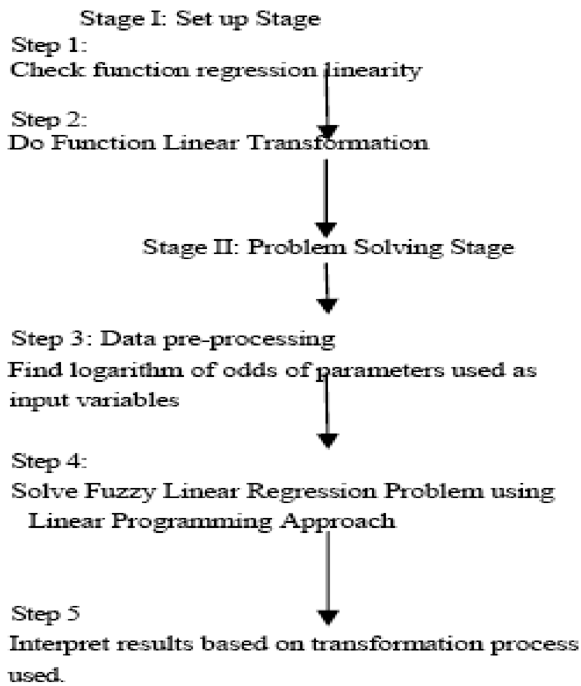
$$\frac{P}{1-P} = e^{a+b_1x_1+b_2x_2+.....+b_jx_j}$$

$$P = \frac{1}{1+e^{-(a+b_1x_1+b_2x_2+.....+b_jx_j)}}$$

where P is the probability of a 1, *e* is the base of the natural logarithm (about 2.718) and *a* and *b* are the parameters of the model.

3.2. The adaptive fuzzy logistic regression model

The adaptive fuzzy logistic regression model is based on Tanaka's possibilistic regression analysis described above in which the response variable Y is written as Y=A0x0 + A1x1 + A2x2 +…..+ Ajxj +…+ Akxk where Y is the fuzzy output, $x = [x1, x2, . . ., xk]T$ is the real-valued input vector of independent variables and each regression coefficient *Aj*, *j*=0, . . ., *k*, was assumed to be a symmetric triangular fuzzy number with center á*j* and half-width *cj*, C*j* ≥0 [3,4]. Tanaka's possibilistic fuzzy regression technique is however applicable to linear functions only [4]. Due to the fact that binary response variable defies the linearity functional relationship that must be satisfied, suitable transformation involving logit (logarithm of odds) transformation must be carried out to unfold the hidden linear relationship. Data must be pre-processed before being fed into a possibilistic fuzzy linear regression model which is then solved by using linear programming to produce a set of corresponding output in an interval form. The output represents the logarithm of the odds for the event to occur. Finally the output is transformed back into the probability of the event occurring by inversing the logarithm of the **odds (logit)** values. In the algorithm presented here it is assumed that the logarithm of the **odds (logit)** is linearly related to X's, the independent variables after undergoing the logit transformation.The algorithm for the adaptive model is summarized in the diagram below:



3.3. The Mean Demand Heuristic

The Mean Demand Heuristic (MDH) is a problem-specific heuristic that provides us with a good starting point for simulated annealing and also provides a lower bound on the search region for the neuro-response surface method. The heuristic can be explained as follows. Let T denote the average cycle time, i.e., the time required to go around the route once and return to the retailer. Let di denote the average demand per customer at the ith retailer. Then, the optimal quantity Qi for the ith retailer, according to this heuristic, is given by $Qi = Tdi\lambda i$, where λi denotes the mean rate of arrival of customers at the ith retailer.

4. COMPUTATIONAL RESULTS

When we compare the performance of the Adaptive Fuzzy Linear Regression Model (AFLRM) to that to the above formulate retailer network problem and compare with the NRSM and MDH. We ran the three methods on a system with two retailers (See Fig 3). We found that that AFLRM is faster and optimal for solving such problems. The main focus of this paper is the optimization of Neuro-Response Surface Method which is replace by Adaptive Fuzzy Logistic Regression Model. AFLRM has the following advantages over other two methods (i.e. NRSM and MDH):

1. In AFLRM, there is no need to assign any weights from input node to the hidden node, table -1 shows the numerical value of both method which is approximately equal to 1.

2. No hidden parameters are there, which cause probabilistic situation and ultimately helps to maximize the accuracy of the results.
3. AFLRM make the situation simple and can be extend to the large problem of Retail Networks.

Table-1

<table>
<tr><td colspan="2">Neuro-Response Surface Method</td><td colspan="5">Neuro-Response Surface Method</td></tr>
<tr><td>w(i,h) (i = 3)</td><td>2,3,4</td><td>Para-meters</td><td>Value</td><td>Para-meters</td><td colspan="2">value</td></tr>
<tr><td>$u_p(i)$</td><td>10,12,15</td><td>b_1</td><td>2</td><td>x_1</td><td>10</td><td rowspan="3">Taking a = 0 P = 1 (appr.)</td></tr>
<tr><td>$V_p^*(h) = \sum_{i=1}^{I} w(i,h) u_p(i)$</td><td>116</td><td>$b_2$</td><td>3</td><td>$x_2$</td><td>12</td></tr>
<tr><td>$V_p(h) = \frac{1}{1+e^{-V_p*(h)}}$ (Value of Sigmoid function)</td><td>1/(1+4.18639E-51) = 1 (appr.)</td><td>b_3</td><td>4</td><td>x_3</td><td>15</td></tr>
</table>

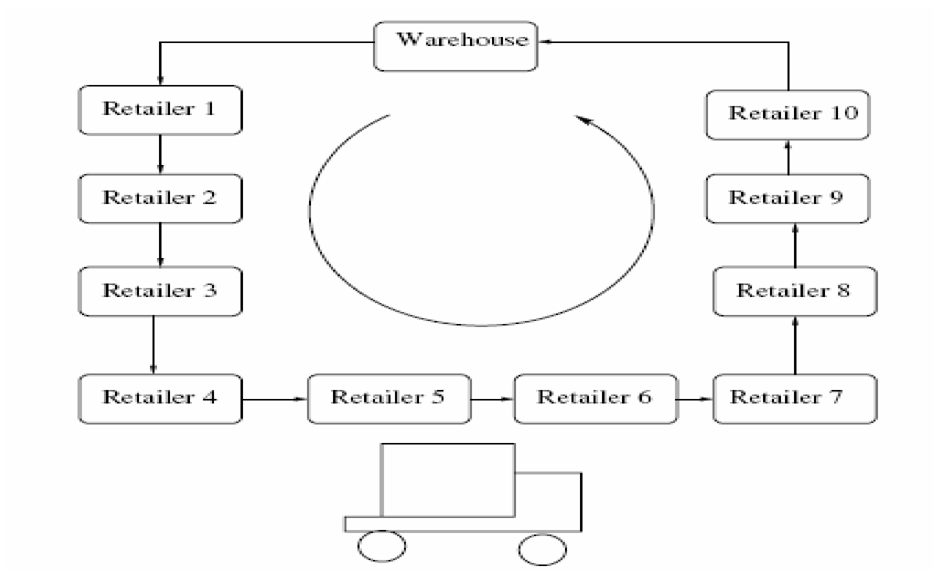


Figure:3

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Received 2009