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A STUDY OF GENERALIZED FRACTIONAL DERIVATIVE OPERATOR INVOLVING MULTIVARIABLE H- FUNCTION AND GENERAL CLASS OF MULTIVARIABLE POLYNOMIALS

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Abstract

In this paper we first establish a generalized fractional derivative operator formula involving a general multivariable polynomial and multivariable H- function. On account of the most general nature of the polynomial and function occurring in our main findings a large number of fractional derivative formulas involving simpler polynomials and functions can be obtained as simple special cases of our main result. Further on reducing our generalized fractional derivative operator to well known Riemann-Liouville fractional derivative operator we get Riemann-Liouville fractional derivative operators involving product of several polynomials and functions. We give exact reference of four authors whose findings follow as special cases of our results. In the process we have also obtained some interesting special cases of our main result that are also believed to be new.

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1. INTRODUCTION

The multivariable H- function due to Srivastava and Panda [14] is defined and represented as follows:

$$H\left[z_1,...,z_r\right]$$

$$= H^{0,N:M_1,N_1;...;M_r,N_r}_{P,Q:P_1,Q_1;...;P_r,Q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,P} : (c_j^{(1)},\gamma_j^{(1)})_{1,P_1};...;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r} \\ (b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,Q} : (d_j^{(1)},\delta_j^{(1)})_{1,Q_1};...;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\psi(s_1,...,s_r)\phi_1(s_1)...\phi_r(s_r)\, z^{s_1}...z^{s_r}\, ds_1...ds_r, \qquad \left(i=\sqrt{-1}\right) \tag{1}$$

where $$\phi_i(s_i) = \frac{\prod_{j=1}^{M_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{N_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=M_i+1}^{Q_i}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=N_i+1}^{P_i}\Gamma(c_j^{(i)}-\gamma_j^{(i)}s_i)} \quad \forall i\in\{1,...,r\}, \tag{2}$$

$$\psi(s_1,...,s_r)=\frac{\prod_{j=1}^{N}\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i)}{\prod_{j=M+1}^{Q}\Gamma(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=N+1}^{P}\Gamma(\alpha_j-\sum_{i=1}^{r}\beta_j^{(i)}s_i)} \quad . \tag{3}$$

For the nature of contours, various sets of convergence conditions of the integral given by (1) and the other details about this function we may refer to [16].

An interesting special case of the multivariable H-function is Generalized Lauricella function given by Srivastava and Daoust [9, p.454] in the following form

$$F_{C:D^{(1)};...;D^{(r)}}^{A:B^{(1)};...;B^{(r)}}\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix}$$

$$=F_{C:D^{(1)};...;D^{(r)}}^{A:B^{(1)};...;B^{(r)}}\left(\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}\left[(a):\theta^{(1)},...,\theta^{(r)}\right]:\left[\left(b^{(1)}\right):\phi^{(1)}\right];...;\left[\left(b^{(r)}\right):\phi^{(r)}\right] \\ \left[(c):\psi^{(1)},...,\psi^{(r)}\right]\left[\left(d^{(1)}\right):\delta^{(1)}\right];...;\left[\left(d^{(r)}\right):\delta^{(r)}\right]\end{matrix}\right.\right)$$

$$=\sum_{m_1,...,m_r=0}^{\infty}\frac{\prod_{j=1}^{A}\left(a_j\right)_{m_1\theta_1^{(1)}+...+m_1\theta_j^{(r)}}\prod_{j=1}^{B^{(1)}}\left(b_j^{(1)}\right)_{m_1\phi_1^{(1)}}\cdots\prod_{j=1}^{B^{(r)}}\left(b_j^{(r)}\right)_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{C}\left(c_j\right)_{m_1\psi_1^{(1)}+...+m_j\psi_j^{(r)}}\prod_{j=1}^{D^{(1)}}\left(d_j^{(1)}\right)_{m_1\delta_1^{(1)}}\cdots\prod_{j=1}^{D^{(r)}}\left(d_j^{(r)}\right)_{m_r\delta_j^{(r)}}}\frac{x_1^{m_1}}{m_1!}\cdots\frac{x_r^{m_r}}{m_r!}\,. \tag{4}$$

and the coefficients

$\theta_j^{(k)}, j=1,...,A;\phi_j^{(k)}, j=1,...,B^{(k)};\psi_j^{(k)}, j=1,...,C^{(k)};\delta_j^{(k)}, j=1,...,D^{(k)};$

$\forall\ k\in\{1,...,r\}$ are real and positive.

When all the coefficients given above are equated to 1 and $B^{(k)},D^{(k)}=0 \quad (k=1,...,r)$ and using the known result [13, p.39, eq. 30], Generalized Lauricella function reduces to generalized hypergeometric function of one variable defined as

$${}_pF_q\left[\begin{matrix}\alpha_1,...,\alpha_p \\ \beta_1,...,\beta_q\end{matrix};x\right]=1+\sum_{n=1}^{\infty}\frac{\prod_{i=1}^{p}(\alpha_i)_n x^n}{\prod_{i=1}^{q}(\beta_i)_n\, n!}\,. \tag{5}$$

where p and q are non-negative integers and no denominator parameter is zero or negative integer.

The General multivariable polynomial, $S_V^{U_1,\ldots,U_k}\left[x_1,\ldots,x_k\right]$ introduced by Srivastava and Garg [11, p.686, eq. (1.4)] is defined in the following manner:

$$S_V^{U_1,\ldots,U_k}\left[x_1,\ldots,x_k\right] = \sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k} U_i R_i \le V} (-V)_{\sum_{i=1}^{k} U_i R_i} A\left(V,R_1,\ldots,R_k\right)\frac{{x_1}^{R_1}}{R_1!}\cdots\frac{{x_k}^{R_k}}{R_k!} \tag{6}$$

where $U_1,\ldots,U_k$ are arbitrary positive integers, V=0,1,2,… and the coefficients $A(V,R_1,\ldots,R_k)$ are arbitrary constants, real or complex. By suitably specializing the coefficients $A(V,R_1,\ldots,R_k)$ occurring in (6) the general multivariable polynomial can be reduced to several multivariable polynomials. Evidently the case k=1 of the polynomial (6) would correspond to the polynomial $S_V^U\left[x\right]$ given by Srivastava [10].

Let $0 \le \alpha < 1, \beta, \eta,\ x \in \Re,\ m \in N$ then the generalized modified fractional derivative operator due to Saigo [8] is defined as

$$D_{0,x,m}^{\alpha,\beta,\eta}\left\{f\right\} = \frac{d}{dz}\left(\frac{z^{m(\beta-\eta)}}{\Gamma(1-\alpha)}\int_0^x \left(x^m - t^m\right)^{-\alpha} {}_2F_1\left[\begin{matrix}\beta-\alpha; 1-\eta; \\ 1-\alpha;\end{matrix} 1-\frac{t^m}{x^m}\right] f\left(t\right) dt^m\right) \tag{7}$$

The multiplicity of $\left(x^m - t^m\right)^{-\alpha}$ in equation (7) is removed by requiring $\log\left(x^m - t^m\right)$ to be real when $\left(x^m - t^m\right) > 0$, and is assumed to be well defined in the unit disk. When m=1 then the above operator reduces to Saigo derivative operator $D_{0,x,}^{\alpha,\beta,\eta}$.

The operator defined by equation (7) includes the well known Riemann-Liouville fractional derivative operator of fractional calculus. Indeed, on putting $\alpha = \beta$ and $m = 1$ we have

$$D_{0,x,}^{\alpha,\alpha,\eta} f\left(x\right) = D_x^{\alpha} f\left(x\right).$$

where D_x^{α} is the familiar Riemann-Liouville fractional derivative operator defined by Miller and Ross [6].

If $0 \le \alpha < 1, m \in N, \beta, \eta, x \in \Re, k > \max\left\{0, m\left(\beta-\eta\right)\right\} - m$, then

$$D_{0,x,m}^{\alpha,\beta,\eta} x^k = \frac{\Gamma\left(1+\frac{k}{m}\right)\Gamma\left(1+\eta-\beta+\frac{k}{m}\right)}{\Gamma\left(1-\beta+\frac{k}{m}\right)\Gamma\left(1+\eta-\alpha+\frac{k}{m}\right)} x^{k-m\beta}, \quad (8)$$

is given by Bhatt and Raina [1].

Some recent results on multivariable H function and fractional integral operators for $\overline{H}$ function by the authors can be found in [2, 3].

2. FRACTIONAL DERIVATIVE FORMULAS

THEOREM 2.1. Let $0 \le \alpha < 1, \beta, \eta, x \in \Re, \theta \in N, \mathrm{Re}(\alpha) > 0, \omega, \delta > 0, \rho_j, \sigma_j > 0 (j = 1, ..., k)$, $\lambda_j, \mu_j > 0 (j = 1, ..., r)$ and $Z_j \in C (j = 1, ..., r)$. If the existence conditions of multivariable H function are satisfied, then the generalized fractional derivative $D_{0,x,\theta}^{\alpha,\beta,\eta}$ of the product of multivariable H function and $S_V^{U_1,...,U_k}(.)$ exists and we have

$$D_{0,x,\theta}^{\alpha,\beta,\eta}\left[t^{\omega}(t+\xi)^{-\delta} S_V^{U_1,...,U_k}\left\{Y_1 t^{\rho_1}(t+\xi)^{-\sigma_1}, ..., Y_k t^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$
$$\left. H\left\{Z_1 t^{\lambda_1}(t+\xi)^{-\mu_1}, ..., Z_r t^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$

$$= x^{\omega-\theta\beta}\xi^{-\delta} \sum_{R_1,...,R_k=0}^{\sum_{i=1}^{k} U_i R_i \le V} (-V)_{\sum_{i=1}^{k} U_i R_i} A(V, R_1, ..., R_k) \frac{Y_1^{R_1}}{R_1!} \cdots \frac{Y_k^{R_k}}{R_k!} \xi^{-\sum_{i=1}^{k}\sigma_i R_i} x^{\sum_{i=1}^{k}\rho_i R_i} \sum_{m=0}^{\infty} \frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$

$$H_{p+3,q+3:p_1,q_1;...;p_r,q_r}^{0,n+3:m_1,n_1;...;m_r,n_r}\left[\begin{array}{l}\left(1-m-\delta-\sum_{i=1}^{k}\sigma_i R_i, \mu_1, ..., \mu_r\right), \left(\frac{-\omega-m-\sum_{i=1}^{k}\rho_i R_i}{\theta}, \frac{\lambda_1}{\theta}, ..., \frac{\lambda_r}{\theta}\right), \\ \left(1-\delta-\sum_{i=1}^{k}\sigma_i R_i, \mu_1, ..., \mu_r\right), \left(\beta-\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}, \frac{\lambda_1}{\theta}, ..., \frac{\lambda_r}{\theta}\right),\end{array}\right.$$

$$\left(\beta-\eta-\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),$$

$$\left(\alpha-\eta-\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),$$

$$\begin{matrix}(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\\(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\left|\begin{matrix}Z_1\xi^{-\mu_1}x^{\lambda_1}\\ \vdots\\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\right] \qquad (9)$$

Proof. By virtue of the results from equation (1) and (6), we have

$$D_{0,x,\theta}^{\alpha,\beta,\eta}\left[t^{\omega}(t+\xi)^{-\delta}S_V^{U_1,\ldots,U_k}\left\{Y_1t^{\rho_1}(t+\xi)^{-\sigma_1},\ldots,Y_kt^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$
$$\left.H\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},\ldots,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$=\left(D_{0,x,\theta}^{\alpha,\beta,\eta}\left[t^{\omega}(t+\xi)^{-\delta}\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V,R_1,\ldots,R_k)\frac{Y_1^{R_1}}{R_1!}\cdots\frac{Y_k^{R_k}}{R_k!}t^{\sum_{i=1}^{k}\rho_iR_i}(t+\xi)^{-\sum_{i=1}^{k}\sigma_iR_i}\right.\right.$$
$$\left.\left.\frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\ldots,s_r)\phi_1(s_1)\ldots\phi_r(s_r)\,Z_1^{s_1}\ldots Z_r^{s_r}t^{\sum_{i=1}^{r}\lambda_is_i}(t+\xi)^{-\sum_{i=1}^{r}\mu_is_i}ds_1\ldots ds_r\right]\right)(x)$$

Interchanging the order of integration and summation and expanding the binomial terms like

$$(t+\xi)^{-l}=\xi^{-l}\sum_{m=0}^{\infty}\frac{(l)_m}{m!}\left(-\frac{t}{\xi}\right)^m,\quad\left|\frac{t}{\xi}\right|<1. \qquad (10)$$

Further using the result (8), we obtain the required result. The interchange of the order of integration and summation is justified under the conditions given in Theorem 1.
When we put $\beta=\alpha$ in equation (9), then we obtain the following result:

COROLLARY 1. If
$\alpha,\omega\in\Re,\theta\in\mathrm{N},\mathrm{Re}(\alpha)>0,\omega,\delta>0,\rho_j,\sigma_j>0\,(j=1,\ldots,k)$,
$\lambda_j,\mu_j>0\,(j=1,\ldots,r)$ and $Z_j\in C\,(j=1,\ldots,r)$. Further let the existence conditions of multivariable H function are satisfied, then the following result holds:

$$D^{\alpha,}_{0,x,\theta}\left[t^{\omega}(t+\xi)^{-\delta} S_V^{U_1,\ldots,U_k}\left\{Y_1t^{\rho_1}(t+\xi)^{-\sigma_1},\ldots,Y_kt^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$
$$\left.H\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},\ldots,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$=x^{\omega-\theta\alpha}\xi^{-\delta}\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V,R_1,\ldots,R_k)\frac{Y_1^{R_1}}{R_1!}\cdots\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_iR_i}x^{\sum_{i=1}^{k}\rho_iR_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$
$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+2:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}\left(1-m-\delta-\sum_{i=1}^{k}\sigma_iR_i,\mu_1,\ldots,\mu_r\right),\left(\frac{-\omega-m-\sum_{i=1}^{k}\rho_iR_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),\\ \left(1-\delta-\sum_{i=1}^{k}\sigma_iR_i,\mu_1,\ldots,\mu_r\right),\left(\alpha-\frac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\\(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\right|\begin{matrix}Z_1\xi^{-\mu_1}x^{\lambda_1}\\ \vdots\\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\right] \quad (11)$$

If we take k=1, then the polynomial $S_V^{U_1,\ldots,U_k}(x_1,\ldots,x_k)$ reduces to $S_V^U(x)$ and it yields

$$D^{\alpha,}_{0,x,\theta}\left[t^{\omega}(t+\xi)^{-\delta} S_V^{U}\left\{Yt^{\rho}(t+\xi)^{-\sigma}\right\}H\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},\ldots,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$=x^{\omega-\theta\alpha}\xi^{-\delta}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}Y^R\xi^{-\sigma R}x^{\rho R}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+2:p_1,q_1;\ldots;p_r,q_r}$$
$$\left[\begin{matrix}Z_1\xi^{-\mu_1}x^{\lambda_1}\\ \vdots\\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\left|\begin{matrix}(1-m-\delta-\sigma R,\mu_1,\ldots,\mu_r),\left(\frac{-\omega-m-\rho R}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),\\ (1-\delta-\sigma R,\mu_1,\ldots,\mu_r),\left(\alpha-\frac{\omega+m+\rho R}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\\(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\right] \quad (12)$$

COROLLARY 2: Let
$0\le\alpha<1,\beta,\eta,x\in\Re,\theta\in N,\mathrm{Re}(\alpha)>0,\omega,\delta>0,\rho_j,\sigma_j>0\,(j=1,\ldots,k),$

$\lambda_j, \mu_j > 0 (j = 1, ..., r)$ and $Z_j \in C (j = 1, ..., r)$. If the existence conditions of multivariable H function are satisfied, then the Saigo derivative operator $D_{0,x}^{\alpha,\beta,\eta}$ of the product of multivariable H function and $S_V^{U_1,...,U_k}(.)$ exists and there holds the following formula

$$D_{0,x}^{\alpha,\beta,\eta}\left[t^{\omega}(t+\xi)^{-\delta} S_V^{U_1,...,U_k}\left\{Y_1 t^{\rho_1}(t+\xi)^{-\sigma_1},...,Y_k t^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$
$$\left.H\left\{Z_1 t^{\lambda_1}(t+\xi)^{-\mu_1},...,Z_r t^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$= x^{\omega-\beta}\xi^{-\delta}\sum_{R_1,...,R_k=0}^{\sum_{i=1}^{k}U_i R_i \le V}(-V)_{\sum_{i=1}^{k}U_i R_i} A(V,R_1,...,R_k)\frac{Y_1^{R_1}}{R_1!}...\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_i R_i}x^{\sum_{i=1}^{k}\rho_i R_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$

$$H_{p+3,q+3:p_1,q_1;...;p_r,q_r}^{0,n+3:m_1,n_1;...;m_r,n_r}\left[\begin{array}{c} Z_1\xi^{-\mu_1}x^{\lambda_1} \\ \vdots \\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{array}\right| \begin{array}{l}\left(1-m-\delta-\sum_{i=1}^{k}\sigma_i R_i,\mu_1,...,\mu_r\right), \\ \left(1-\delta-\sum_{i=1}^{k}\sigma_i R_i,\mu_1,...,\mu_r\right),\end{array}$$
$$\left(-\omega-m-\sum_{i=1}^{k}\rho_i R_i,\lambda_1,...,\lambda_r\right),\left(\beta-\eta-\omega-m-\sum_{i=1}^{k}\rho_i R_i,\lambda_1,...,\lambda_r\right),$$
$$\left(\beta-\omega-m-\sum_{i=1}^{k}\rho_i R_i,\lambda_1,...,\lambda_r\right),\left(\alpha-\eta-\omega-m-\sum_{i=1}^{k}\rho_i R_i,\lambda_1,...,\lambda_r\right),$$
$$\left.\begin{array}{l}(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};...;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r} \\ (b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};...;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{array}\right] \quad (13)$$

If we take $\beta = \alpha$ in equation (13), then we obtain the following result for the Riemann-Liouville fractional derivative operator defined by Miller and Ross.

COROLLARY 3: If
$\alpha, \omega \in \Re, \mathrm{Re}(\alpha) > 0, \omega, \delta > 0, \rho_j, \sigma_j > 0 (j = 1, ..., k), \lambda_j, \mu_j > 0 (j = 1, ..., r)$
and $Z_j \in C (j = 1, ..., r)$. Further let the existence conditions of multivariable H function are satisfied, then

$$D_{0,x}^{\alpha}\left[t^{\omega}(t+\xi)^{-\delta}S_V^{U_1,\dots,U_k}\left\{Y_1t^{\rho_1}(t+\xi)^{-\sigma_1},\dots,Y_kt^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$
$$\left.H\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},\dots,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$=x^{\omega-\alpha}\xi^{-\delta}\sum_{R_1,\dots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V,R_1,\dots,R_k)\frac{Y_1^{R_1}}{R_1!}\cdots\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_iR_i}x^{\sum_{i=1}^{k}\rho_iR_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$
$$H_{p+2,q+2:p_1,q_1;\dots;p_r,q_r}^{0,n+2:m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix}Z_1\xi^{-\mu_1}x^{\lambda_1}\\ \vdots\\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\middle|\begin{matrix}\left(1-m-\delta-\sum_{i=1}^{k}\sigma_iR_i,\mu_1,\dots,\mu_r\right),\\ \left(1-\delta-\sum_{i=1}^{k}\sigma_iR_i,\mu_1,\dots,\mu_r\right),\end{matrix}\right.$$
$$\left.\begin{matrix}\left(-\omega-m-\sum_{i=1}^{k}\rho_iR_i,\lambda_1,\dots,\lambda_r\right),(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\\ \left(\alpha-\omega-m-\sum_{i=1}^{k}\rho_iR_i,\lambda_1,\dots,\lambda_r\right),(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\right]$$

(14)

Further by taking s=1, polynomial $S_V^{U_1,\dots,U_k}(x_1,\dots,x_k)$ reduces to $S_V^U(x)$ and it yields

$$D_{0,x}^{\alpha}\left[t^{\omega}(t+\xi)^{-\delta}S_V^{U}\left\{Yt^{\rho}(t+\xi)^{-\sigma}\right\}H\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},\dots,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$
$$=x^{\omega-\alpha}\xi^{-\delta}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}Y^R\xi^{-\sigma R}x^{\rho R}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m H_{p+2,q+2:p_1,q_1;\dots;p_r,q_r}^{0,n+2:m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix}Z_1\xi^{-\mu_1}x^{\lambda_1}\\ \vdots\\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\right.$$
$$\begin{matrix}(1-m-\delta-\sigma R,\mu_1,\dots,\mu_r),(-\omega-m-\rho R,\lambda_1,\dots,\lambda_r),\\ (1-\delta-\sigma R,\mu_1,\dots,\mu_r),(\alpha-\omega-m-\rho R,\lambda_1,\dots,\lambda_r),\end{matrix}$$
$$\left.\begin{matrix}(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\\ (b_j;\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,Q}:(d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\right] \qquad (15)$$

If we take $V=0,\ A_{0,1}=1$ in equation (15), then $S_V^U(x)=1$ and replacing $\delta\to-\delta,\mu_j\to-\mu_j(j=1,\dots,r)$ we arrive at the result given by Srivastava and Goyal [12, p.644, eq. (2.1)] after little simplifications. Further putting $\mu_j\to 0,(j=1,\dots,r)$

and $\delta = 0$ we get the result. [12, pp. 647-648, eqs. (3.1), (3.3)], which on further specializations would yield several known results due to Raina and Koul [7], Manocha and Sharma [4, 5] and others.

3. FRACTIONAL DERIVATIVE FORMULA OF THE GENERALIZED LAURICELLA FUNCTION AND GENERALIZED HYPERGEOMETRIC FUNCTION

In this section we represent fractional derivative formulas of the Generalized Lauricella function and Generalized Hypergeometric function defined in (4) and (5). We deduce the following formulas from Theorem 1 and its sub theorems for the fractional derivative operator of $F_{C:D';...;D^{(n)}}^{A:B';...;B^{(n)}}\left[z_1 \quad \dots \quad z_n\right]$ and ${}_PF_Q\left(z\right)$.

COROLLARY 4: Let $0 \le \alpha < 1, \beta, \eta, \mathrm{x} \in \Re, \theta \in \mathrm{N}, \mathrm{Re}(\alpha) > 0, \omega, \delta > 0.$ Further let the constants $\rho_{\mathrm{j}}, \sigma_{\mathrm{j}} > 0\left(\mathrm{j}=1,...,\mathrm{k}\right), \lambda_{\mathrm{j}}, \mu_{\mathrm{j}} > 0\left(\mathrm{j}=1,...,\mathrm{r}\right)$ and $\mathrm{Z}_{\mathrm{j}} \in \mathrm{C}\left(\mathrm{j}=1,...,\mathrm{r}\right)$. If the conditions of (9) are satisfied, then the generalized fractional derivative $D_{0,x,\theta}^{\alpha,\beta,\eta}$ of the product of Lauricella function and $\mathrm{S}_{\mathrm{V}}^{\mathrm{U}_1,...,\mathrm{U}_\mathrm{k}}(.)$ exists and there holds the following formula

$$D_{0,x,\theta}^{\alpha,\beta,\eta}\left[t^{\omega}\left(t+\xi\right)^{-\delta} S_V^{U_1,...,U_k}\left\{Y_1 t^{\rho_1}\left(t+\xi\right)^{-\sigma_1},...,Y_k t^{\rho_k}\left(t+\xi\right)^{-\sigma_k}\right\}\right.$$
$$\left. F_{Q:Q_1;...;Q_r}^{P:P_1;...;P_r}\left\{Z_1 t^{\lambda_1}\left(t+\xi\right)^{-\mu_1},...,Z_r t^{\lambda_r}\left(t+\xi\right)^{-\mu_r}\right\}\right](x)$$
$$= x^{\omega-\theta\beta}\xi^{-\delta}\sum_{R_1,...,R_k=0}^{\sum_{i=1}^{k}U_iR_i \le V}(-V)_{\sum_{i=1}^{k}U_iR_i} A\left(V,R_1,...,R_k\right)\frac{Y_1^{R_1}}{R_1!}\cdots\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_iR_i}x^{\sum_{i=1}^{k}\rho_iR_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$
$$\left(\delta+\sum_{i=1}^{k}\sigma_iR_i\right)_m \frac{\Gamma\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)\Gamma\left(1-\beta+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)}{\Gamma\left(1-\beta+\frac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)\Gamma\left(1-\alpha+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)}$$

$$F^{P+3:P_1;...;P_r}_{Q+3:Q_1;...;Q_r}\left[\begin{matrix} Z_1\xi^{-\mu_1}x^{\lambda_1} \\ \vdots \\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\left|\begin{matrix}\left(m+\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,...,\mu_r\right) \\ \left(\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,...,\mu_r\right)\end{matrix}\right.\right.$$

$$\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},...,\frac{\lambda_r}{\theta}\right),\left(1-\beta+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},...,\frac{\lambda_r}{\theta}_r\right),$$

$$\left(1-\beta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},...,\frac{\lambda_r}{\theta}\right),\left(1-\alpha+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},...,\frac{\lambda_r}{\theta}\right),$$

$$\left.\begin{matrix}(1-a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,P}:(1-c_j^{(1)},\gamma_j^{(1)})_{1,P_1};...;(1-c_j^{(r)},\gamma_j^{(r)})_{1,P_r} \\ (1-b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,Q}:(1-d_j^{(1)},\delta_j^{(1)})_{1,Q_1};...;(1-d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{matrix}\right] \quad (16)$$

If, however, we take $\beta=\alpha$ in equation (16), then we obtain the following result for the well known fractional derivative operator.

COROLLARY 5: If $0\le\alpha<1,\alpha,\omega\in\Re,\theta\in \mathrm{N},\mathrm{Re}(\alpha)>0,\omega,\delta>0.$ Further let the constants $\rho_j,\sigma_j>0(j=1,...,k),\lambda_j,\mu_j>0(j=1,...,r)$ and $Z_j\in C(j=1,...,r)$, then for $x>0$

$$D^{\alpha}_{0,x,\theta}\left[t^{\omega}(t+\xi)^{-\delta}S_V^{U_1,...,U_k}\left\{Y_1t^{\rho_1}(t+\xi)^{-\sigma_1},...,Y_kt^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$

$$\left.F^{P:P_1;...;P_r}_{Q:Q_1;...;Q_r}\left\{Z_1t^{\lambda_1}(t+\xi)^{-\mu_1},...,Z_rt^{\lambda_r}(t+\xi)^{-\mu_r}\right\}\right](x)$$

$$=x^{\omega-\theta\alpha}\xi^{-\delta}\sum_{R_1,...,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}A(V,R_1,...,R_k)\frac{Y_1^{R_1}}{R_1!}...\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_iR_i}x^{\sum_{i=1}^{k}\rho_iR_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$

$$\left(\delta+\sum_{i=1}^{k}\sigma_i R_i\right)_m \frac{\Gamma\left(1+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}{\Gamma\left(1-\alpha+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}$$

$$F_{Q+2:Q_1;\ldots;Q_r}^{P+2:P_1;\ldots;P_r}\left[\begin{array}{c} Z_1\xi^{-\mu_1}x^{\lambda_1} \\ \vdots \\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{array}\left|\begin{array}{l}\left(m+\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,\ldots,\mu_r\right),\left(1+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\dfrac{\lambda_1}{\theta},\ldots,\dfrac{\lambda_r}{\theta_r}\right), \\ \left(\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,\ldots,\mu_r\right),\left(1-\alpha+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\dfrac{\lambda_1}{\theta},\ldots,\dfrac{\lambda_r}{\theta}\right),\end{array}\right.\right.$$

$$\left.\begin{array}{l}(1-a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,P}:(1-c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(1-c_j^{(r)},\gamma_j^{(r)})_{1,P_r} \\ (1-b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,Q}:(1-d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(1-d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\end{array}\right] \tag{17}$$

If we put $U_1=\ldots=U_k=1$ and reduce the polynomial $S_V^{U_1,\ldots,U_k}\left(x_1,\ldots,x_k\right)$ to multivariable Jacobi polynomial defined by Srivastava [14, p.65, eq. (14)] and we obtain

$$D_{0,x,\theta}^{\alpha,}\left[t^{\omega}\left(t+\xi\right)^{-\delta}P_V^{\alpha_1,\beta_1;\ldots;\alpha_k,\beta_k}\left\{1-2Y_1t^{\rho_1}\left(t+\xi\right)^{-\sigma_1},\ldots,1-2Y_kt^{\rho_k}\left(t+\xi\right)^{-\sigma_k}\right\}\right.$$

$$\left.F_{Q:Q_1;\ldots;Q_r}^{P:P_1;\ldots;P_r}\left\{Z_1t^{\lambda_1}\left(t+\xi\right)^{-\mu_1},\ldots,Z_rt^{\lambda_r}\left(t+\xi\right)^{-\mu_r}\right\}\right](x)$$

$$=x^{\omega-\theta\alpha}\xi^{-\delta}\frac{\prod_{i=1}^{k}(1+\alpha_i)_V}{(V!)^k}\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}R_i\le V}(-V)_{\sum_{i=1}^{k}R_i}\prod_{i=1}^{k}\frac{(1+\alpha_i+\beta_i+V)_{R_i}}{(1+\alpha_i)_{R_i}R_i!}Y_i^{R_i}$$

$$\xi^{-\sum_{i=1}^{k}\sigma_i R_i} x^{\sum_{i=1}^{k}\rho_i R_i} \sum_{m=0}^{\infty} \frac{1}{m!}\left(-\frac{x}{\xi}\right)^m \left(\delta+\sum_{i=1}^{k}\sigma_i R_i\right)_m \frac{\Gamma\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}{\Gamma\left(1-\alpha+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}$$

$$F_{Q+2:Q_1;\ldots;Q_r}^{P+2:P_1;\ldots;P_r}\left[\begin{matrix} Z_1\xi^{-\mu_1}x^{\lambda_1} \\ \vdots \\ Z_r\xi^{-\mu_r}x^{\lambda_r}\end{matrix}\middle| \begin{matrix} \left(m+\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,\ldots,\mu_r\right),\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right), \\ \left(\delta+\sum_{i=1}^{k}\sigma_i R_i,\mu_1,\ldots,\mu_r\right),\left(1-\alpha+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta},\frac{\lambda_1}{\theta},\ldots,\frac{\lambda_r}{\theta}\right), \end{matrix}\right.$$

$$\left.\begin{matrix} (1-a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,P}:(1-c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(1-c_j^{(r)},\gamma_j^{(r)})_{1,P_r} \\ (1-b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,Q}:(1-d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(1-d_j^{(r)},\delta_j^{(r)})_{1,Q_r} \end{matrix}\right] \quad (18)$$

COROLLARY 6: Let $0 \le \alpha < 1, \beta, \eta, x \in \Re, \theta \in N, \mathrm{Re}(\alpha) > 0, \omega, \delta > 0.$ Further let the constants $\rho_j, \sigma_j > 0\,(j=1,\ldots,k), \lambda_j, \mu_j > 0\,(j=1,\ldots,r)$ and $Z_j \in C\,(j=1,\ldots,r)$. If the conditions of (9) are satisfied, then the generalized fractional derivative operator $D_{0,x,\theta}^{\alpha,\beta,\eta}$ of the product of generalized hyper geometric function and $S_V^{U_1,\ldots,U_k}(.)$ exists and we get the following formula

$$D_{0,x,\theta}^{\alpha,\beta,\eta}\left[t^{\omega}(t+\xi)^{-\delta} S_V^{U_1,\ldots,U_k}\left\{Y_1 t^{\rho_1}(t+\xi)^{-\sigma_1},\ldots,Y_k t^{\rho_k}(t+\xi)^{-\sigma_k}\right\}\right.$$

$$\left.{}_pF_q\left\{(Z_1+\ldots+Z_r)t^{\lambda}(t+\xi)^{-\mu}\right\}\right](x)$$

$$= x^{\omega-\theta\beta}\xi^{-\delta}\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i} A(V,R_1,\ldots,R_k)\frac{Y_1^{R_1}}{R_1!}\cdots\frac{Y_k^{R_k}}{R_k!}\xi^{-\sum_{i=1}^{k}\sigma_i R_i} x^{\sum_{i=1}^{k}\rho_i R_i}\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m$$

$$\left(\delta+\sum_{i=1}^{k}\sigma_i R_i\right)_m \frac{\Gamma\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)\Gamma\left(1-\beta+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}{\Gamma\left(1-\beta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)\Gamma\left(1-\alpha+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right)}$$

$$ {}_{P+3}F_{Q+3}\left[\begin{matrix}\left(m+\delta+\sum_{i=1}^{k}\sigma_i R_i\right),\left(1+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right),\left(1-\beta+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right),(a_P)\\ \left(\delta+\sum_{i=1}^{k}\sigma_i R_i\right),\left(1-\beta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right),\left(1-\alpha+\eta+\frac{\omega+m+\sum_{i=1}^{k}\rho_i R_i}{\theta}\right),(b_Q)\end{matrix}\middle|\left(Z_1+\ldots+Z_r\right)\xi^{-\mu}x^{\lambda}\right] \tag{19}$$

Further letting $\beta=\alpha$ in equation (19), we get the following result

$$D_{0,x,\theta}^{\alpha}\left[t^{\omega}\left(t+\xi\right)^{-\delta} S_V^{U_1,\ldots,U_k}\left\{Y_1 t^{\rho_1}\left(t+\xi\right)^{-\sigma_1},\ldots,Y_k t^{\rho_k}\left(t+\xi\right)^{-\sigma_k}\right\}\right.$$
$$\left.{}_PF_Q\left\{\left(Z_1+\ldots+Z_r\right)t^{\lambda}\left(t+\xi\right)^{-\mu}\right\}\right](x)$$

$$= x^{\omega-\theta\alpha}\xi^{-\delta}\sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V}(-V)_{\sum_{i=1}^{k}U_iR_i}\,A(V,R_1,\ldots,R_k)\frac{Y_1^{R_1}}{R_1\,!}\cdots\frac{Y_k^{R_k}}{R_k\,!}\xi^{-\sum_{i=1}^{k}\sigma_iR_i}x^{\sum_{i=1}^{k}\rho_iR_i}$$

$$\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{x}{\xi}\right)^m\left(\delta+\sum_{i=1}^{k}\sigma_iR_i\right)_m\frac{\Gamma\left(1+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)}{\Gamma\left(1-\alpha+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)}$$

$${}_{P+2}F_{Q+2}\left[\begin{array}{c|c}\left(m+\delta+\sum_{i=1}^{k}\sigma_iR_i\right)\left(1+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)(a_P) & \\ & (Z_1+\ldots+Z_r)\xi^{-\mu}x^{\lambda}\\ \left(\delta+\sum_{i=1}^{k}\sigma_iR_i\right)\left(1-\alpha+\dfrac{\omega+m+\sum_{i=1}^{k}\rho_iR_i}{\theta}\right)(b_Q) & \end{array}\right]$$

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