Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), **7** (2011), No. 2

FUNCTION AND GENERAL CLASS OF POLYNOMIAL AND DISPLACEMENT OF THE VIBRATING STRING

S.C.SHARMA AND RACHNA BHARGAVA

Abstract

In this paper, first we evaluate finite integral involving products of general class of polynomials and $\overline{H}$-functions. Next we make its application to solve a boundary value problem on deflection of vibrating string under certain conditions. In view of generality of the polynomials and product of $\overline{H}$-function occurring herein, on specializing the coefficients of polynomials and parameters of the $\overline{H}$-function, our results would readily reduce to several simpler results.

Mathematics Subject Classification 2000: 33C60, 34B05

Additional Key Words and Phrases: General Class of Polynomials, $\overline{H}$-function.

1. INTRODUCTION

The general class of polynomials introduced by Srivastava [6] will be defined and represented as:

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{m,k}}{k!} A_{n,k}\, x^k,\ n = 0,1,2,... \quad (1.1)$$

Where m is an arbitrary positive integer the coefficient $A_{n,k}$ $(n,k \geq 0)$ are arbitrary constant, real or complex.

The $\overline{H}$-function occurring in the paper is defined and represented as follows [3] and [1].

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi\omega}\int_L \overline{\phi}(\xi)\, z^{\xi}\, d\xi,\ \omega = \sqrt{-1} \quad (1.2)$$

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.3}$$

Here $a_j(j=1,...,P)$ and $b_j(j=1,...,Q)$ are complex parameters $\alpha_j(j=1,...,P)$; $\beta_j(j=1,...,Q), A_j(j=1,...,N); B_j(j=M+1,...,Q)$ are non-negative real numbers. The sufficient conditions for the absolute convergence of the integral have been established by Bushman and Srivastava in the paper referred above. It is assumed that the corresponding conditions are satisfied appropriately by all $\bar{H}$-function occurring through out the paper, when all the exponents A_j and B_j take the value 1, the $\bar{H}$-function reduces to the well known Fox H-function.

The following series representation for the $\bar{H}$-function was obtained by Rathie [5]:

$$\bar{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N},(c_j,\gamma_j)_{N+1,P}\\(d_j,\delta_j)_{1,M},(d_j,\delta_j;D_j)_{M+1,Q}\end{matrix}\right.\right]=\sum_{r=0}^{\infty}\sum_{h=1}^{M}\bar{\theta}(\xi_{h,r})z^{\xi_{h,r}} \tag{1.4}$$

Where

$$\bar{\theta}(\xi_{h,r})=\frac{\prod_{j=1,\,j\neq 1}^{M}\Gamma(d_j-\delta_j\xi_{h,r})\prod_{j=1}^{N}\{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j}\prod_{h=N+1}^{P}\Gamma(c_j-\gamma_j\xi_{h,r})}\frac{(-1)^r}{r!\delta_h}\qquad \xi_{h,r}=\frac{d_r+h}{\delta_r} \tag{1.5}$$

2. MAIN INTEGRAL

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{u-1}\sin\frac{\lambda_m\pi x}{L}S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right]S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\overline{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\(d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]dx$$

$$=k\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{M_2}z^{-(b_1k_1+b_2k_2+h_2\xi_{h_2,r})}\frac{(-n_1)_{m_1,k_1}A_{n_1,k_1}}{k_1!}y_1^{k_1}$$

$$\frac{(-n_2)_{m_2,k_2},A_{n_2k_2}}{k_2!}y_2^{k_2}\sin\frac{\pi\lambda_m}{2}\overline{\theta}(\xi_{h_2,r})z_2^{\xi_{h_2,r}}$$

$$\overline{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^{h_1}}\right)\left|\begin{matrix}(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r},h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}.(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right] \quad (2.1)$$

Where

(i) $k=L.2^{-u+1}$

(ii) $(h_1,h_2,n_1,n_2,k_1,k_2)>0$ and provided that conditions

(i) $\min \mathrm{Re}\,(b_j/\beta_j)>0$ and $\xi=\dfrac{b_h+r}{\beta_h}$

(ii) $\min \mathrm{Re}\,(d_j/\delta_j)>0$ and $\xi_{h,r}=\dfrac{d_h+r}{\delta_h}$

(iii) $\mathrm{Re}\,(u+b_1k_1+b_2k_2+h_2\xi_{h_1r}+h_1\xi)>0$

Proof: To establish the above integral (2.1), we first express both the general class of polynomial $\overline{H}^{M_1,N_1}_{P_1,Q_1}[z_1]$ and $\overline{H}^{M_2,N_2}_{P_2,Q_2}[z]$ occurring in its left hand side in their respective contour integral and series form with the help of equations (1.1), (1.2) and (1.4) respectively and then interchange the order of integration and summation which is permissible under the condition) denoting the expression thus obtained by I_1, we get

$$I_1=\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h=1}^{M_2}\frac{(-n_1)_{m_1,k_1}A_{n_1,k_1}}{k_1!}y_1^{k_1}$$

$$\frac{(-n_2)_{m_2,k_2}A_{n_2,k_2}}{k_2!}y_2^{k_2}z_2^{\xi_{h_2,r}}$$

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{+\omega\infty}\bar{\phi}(\xi)z_1^{\xi}\left\{\int_0^L\left(\sin\frac{\pi x}{L}\right)^{u+b_1k_1+b_2k_2+h_2\xi h_2,r+h_1\xi-1}\left(\sin\frac{\lambda_m\pi x}{L}\right)dx\right\}d\xi \tag{2.2}$$

Evaluating the x-integral involved in (2.2) with the help of result [2, p.371, Eq.(1)]

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{u-1}\sin\left(\frac{\lambda_m\pi x}{L}\right)dx=\frac{\Gamma(u)\sin\frac{\pi\lambda_m}{2}}{2^{u-1}\Gamma\left(\frac{u\pm\lambda_m+1}{2}\right)}$$

Where Re(u) > 0 and $\Gamma(a \pm b)$ represents Γ (a+b). Γ (a–b).

Now if we reinterpret the result thus obtained with the help of equations (1.2) and (1.3) in terms of the $\overline{H}$-function, we easily arrive the right hand side of (2.1) after a little simplification.

Thus completes the proof.

3. APPLICATION TO HOMOGENEOUS WAVE EQUATION

It is well known that the definition θ (x,t) of vibrating string satisfies the familiar wave equation.

$$\frac{\partial^2\theta}{\partial x^2}=\frac{1}{c}\frac{\partial^2\theta}{\partial t^2},\ 0<x<L,\ t>0 \quad \text{…(3.1)}$$

Now we assume that the boundary conditions

$$\theta(0,t)=0,\ \theta(L,t)=0,\ t>0 \quad \text{…(3.2)}$$

According to Powers [4], the solution of the problem can be written as:

$$\theta(x,t)=\sum_{m=1}^{\infty}\left\{\left(a_m\cos\frac{\lambda_m\pi ct}{L}+b_m\sin\frac{\lambda_m\pi ct}{L}\right)\sin\frac{\lambda_m\pi x}{L}\right\} \tag{3.3}$$

This gives

$$\frac{\partial\theta(x,t)}{\partial t}=\sum_{m=1}^{\infty}\left\{(-a_m\frac{\lambda_m\pi c}{L}\sin\frac{\lambda_m ct}{L}+b_m\frac{\lambda_m\pi c}{L}\cos\frac{\lambda_m\pi ct}{L}\sin\frac{\lambda_m\pi x}{L}\right\}$$

(3.4)

Now we consider the problem of determining $\theta(x,t)$

Where

$$\theta(x,0)=f(x)=\left(\sin\frac{\pi x}{L}\right)^{u-1}S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right]S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\overline{H}_{P_1,Q_1}^{M_1,N_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\overline{H}_{P_2,Q_2}^{M_2,N_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\(d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

(3.5)

For t = 0, (3.3) and (3.4) reduce to

$$\theta(x,0)=\sum_{m=1}^{\infty}a_m\sin\frac{\lambda_m\pi x}{L}$$

(3.6)

and

$$\frac{\partial\theta(x,0)}{\partial t}=\sum_{m=1}^{\infty}\frac{\lambda_m\pi c}{L}b_m\sin\frac{\lambda_m\pi}{L}$$

(3.7)

Respectively, we substituting the value of $\theta(x,0)$ from (3.5) in (3.6) and multiplying both sides of $\sin\frac{\lambda_m\pi x}{L}$ and then integrating with respect to 0 to L, we get

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{u-1}\sin\frac{\lambda_m\pi x}{L}S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right]S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\overline{H}_{P_1,Q_1}^{M_1,N_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\overline{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\(d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$
$$=\int_0^L a_m \sin\frac{\lambda_m \pi x}{L}\sin\frac{\lambda_m \pi x}{L}\,dx \tag{3.8}$$

Now making an appeal to (2.1), and we use orthogonal property of sine function due to [4]

$$\int_0^L \sin\frac{\lambda_m \pi x}{L}\sin\frac{\lambda_p \pi x}{L}\,dx=\begin{cases}0, m\neq p\\ L/2\left(1-\dfrac{1}{2\pi\lambda_m}\sin 2\pi\lambda_m\right), m=p\end{cases}$$

We derive from (3.8)

$$a_m=\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{M_2} z^{-(b_1k_1+b_2k_2+h_2\xi_{h_2,r})}\frac{(-n_1)_{m_1,k_1}A_{n_1,k_1}}{k_1!}y_1^{k_1}$$
$$\frac{(-n_2)_{m_2,k_2},A_{n_2,k_2}}{k_2!}y_2^{k_2}\,\overline{\theta}(\xi_{h_2,r})z_2^{\xi_{h_2,r}}$$
$$\overline{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^{h_1}}\right)\left|\begin{matrix}(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r},h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}.(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right] \tag{3.9}$$

And

$$\frac{\partial\theta(x,0)}{\partial t}=g(x)=\left(\sin\frac{\pi x}{L}\right)^{u'-1}S^{m_1'}_{n_1'}\left[y_1'\left(\sin\frac{\pi x}{L}\right)^{b_1'}\right]S^{m_2'}_{n_2'}\left[y_2'\left(\sin\frac{\pi x}{L}\right)^{b_2'}\right]$$
$$\overline{H}^{M_1',N_1'}_{P_1',Q_1'}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1'}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1'},(a_j,\alpha_j)_{N_1'+1,P_1'}\\(b_j,\beta_j)_{1,M_1'},(b_j,\beta_j;B_j)_{M_1'+1,Q_1'}\end{matrix}\right.\right]$$
$$\overline{H}^{M_2',N_2'}_{P_2',Q_2'}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2'}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2'},(c_j,\gamma_j)_{N_2'+1,P_2'}\\(d_j,\delta_j)_{1,M_2'},(d_j,\delta_j;D_j)_{M_2'+1,Q_2'}\end{matrix}\right.\right]\ 0<x<L \tag{3.10}$$

We further substitute the value of $\frac{\partial\theta(x,0)}{\partial t}$ from (3.10) in (3.7) and applied the method as above after little simplification we get the value for b_m.

$$b_m = k'\sum_{m=1}^{\infty}\sum_{k_1'=0}^{[n_1'/m_1']}\sum_{k_2'=0}^{[n_2'/m_2']}\sum_{r=0}^{\infty}\sum_{h_2'=1}^{M_2'} z^{-(b_1'k_1'+b_2'k_2'+h_2'\xi_{n_2,r})}\frac{(-n_1')_{m_1',k_1'}A_{n_1',k_1'}}{k_1'!}y_1^{'k_1'}$$

$$\frac{(-n_2')_{m_2',k_2'},A_{n_2',k_2'}}{k_2'!}y_2^{'k_2'}\ \overline{\psi}(S_{h_2',r})z_2^{S_{h_2',r}}\frac{\lambda_m\left(\sin\frac{\pi\lambda_m}{2}\right)}{(2\pi\lambda_m-\sin 2\pi\lambda_m)}$$

$$\overline{H}^{M_1',N_1'+1}_{P_1'+1,Q_1'+1}\left[\left(\frac{z_1}{2^{h_1'}}\right)\middle|\begin{matrix}(1-u'-b_1'k_1'-b_2'k_2'-h_2'\xi_{h_2',r},h_1';1),(a_j,\alpha_j;A_j)_{1,N_1'},(a_j,\alpha_j)_{N_1'+1,P_1'}\\(b_j,\beta_j)_{1,M_1'},(b_j,\beta_j;B_j)_{M_1'+1,Q_1'}.(1-u'-b_1'k_1'-b_2'k_2'-h_2'\xi_{h_2',r}\mp\lambda_m,h_1';1)\end{matrix}\right]$$

(3.11)

Now substitute the values of a_m and b_m from (3.9) and (3.11) respectively in (3.3), we easily arrive at the desired result after a little simplification as shown below

$$\theta(x,t) = k'\sum_{m=1}^{\infty}\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{M_2} z^{-(b_1k_1+b_2k_2+h_2\xi_{h_2,r})}\frac{(-n_1)_{m_1,k_1}A_{n_1,k_1}}{k_1!}y_1^{k_1}$$

$$\frac{(-n_2)_{m_2,k_2},A_{n_2,k_2}}{k_2!}y_2^{k_2}\,\overline{\theta}(\xi_{h_2,r})z_2^{\xi_{h_2,r}}\lambda_m\frac{\cos\left(\frac{\lambda_m\pi ct}{L}\right)\sin\left(\frac{\lambda_m\pi x}{L}\right)\left(\sin\frac{\pi\lambda_m}{2}\right)}{(2\pi\lambda_m-\sin 2\pi\lambda_m)}$$

$$\overline{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^{h_1}}\right)\middle|\begin{matrix}(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r},h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\left(\frac{1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r}\mp\lambda_m,h_1;1}{2}\right)\end{matrix}\right]$$

$$+\,k'\sum_{m=1}^{\infty}\sum_{k_1'=0}^{[n_1'/m_1']}\sum_{k_2'=0}^{[n_2'/m_2']}\sum_{r=0}^{\infty}\sum_{h_2'=1}^{M_2'} z^{-(b_1'k_1'+b_2'k_2'+h_2'\xi_{h_2',r})}\frac{(-n_1')_{m_1',k_1'}A_{n_1',k_1'}}{k_1'!}y_1^{'k_1'}$$

$$\frac{(-n_2')_{m_2',k_2'},A_{n_2'k_2'}}{k_2'!}y_2^{'k_2'}\ \overline{\psi}(S_{h_2',r})z_2^{S_{h_2',r}}\frac{\left(\sin\frac{\pi\lambda_m}{L}\right)\left(\sin\frac{\pi\lambda_m ct}{L}\right)}{(2\pi\lambda_m-\sin 2\pi\lambda_m)}\sin\frac{\lambda_m\pi x}{L}$$

$$\bar{H}^{M_1',N_1'+1}_{P_1'+1,Q_1'+1}\left[\left(\frac{z_1}{2^{h_1'}}\right)\middle|\begin{matrix}(1-u'-b_1'k_1'-b_2'k_2'-h_2'\xi_{h_2,r}',h_1';1),(a_j,\alpha_j;A_j)_{1,N_1'},(a_j,\alpha_j)_{N_1'+1,P_1'}\\(b_j,\beta_j)_{1,M_1'},(b_j,\beta_j;B_j)_{M_1'+1,Q_1'}.(1-u'-b_1'k_1'-b_2'k_2'-h_2'\xi_{h_2',r}\mp\lambda_m,h_1';1)\end{matrix}\right] \tag{3.12}$$

Where

$$k' = 2\pi\lambda_m$$

Provided that all conditions of (2.1) are satisfied.

4. SPECIAL CASE

If in equation (1) we reduce $S^{m_1}_{n_1}[x_1]$ to Hermite polynomials [7,p.106, eq.(5.5.4) and $S^{m_2}_{n_2}[x_2]$ to Jacobi polynomials [7, p.68, eq.(4.3.2)], we obtain the following useful and new result is given by

$$\theta(x,t) = k'\sum_{m=1}^{\infty}\sum_{r=0}^{\infty}\sum_{h_2=1}^{M_2} z^{-(b_1k_1+b_2k_2+h_2\xi_{n_2,r})}\, x^{n_1/2}\, H_{n_1}\left(\frac{1}{2\sqrt{\left(\sin\frac{\pi x}{L}\right)^{b_1}}}\right)\binom{n_1+\alpha}{n_1}$$

$$\frac{(\alpha+\beta+n_2+1)}{(\alpha+1)_k}\bar{\theta}(\xi_{h_2r})\, z_2^{\xi_{h_2,r}}\,\frac{\cos\left(\frac{\lambda_m\pi ct}{L}\right)\sin\left(\frac{\lambda_m\pi x}{L}\right)\left(\sin\frac{\pi\lambda_m}{2}\right)}{((2\pi\lambda_m-\sin 2\pi\lambda_m)}$$

$$\bar{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^{h_1}}\right)\middle|\begin{matrix}(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r},h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}.(1-u-b_1k_1-b_2k_2-h_2\xi_{h_2,r}\mp\lambda_m,h_1;1)\end{matrix}\right]$$

$$+k'\sum_{m=1}^{\infty}\sum_{r=0}^{\infty}\sum_{h_2'=1}^{M_2'} z^{-(b_1'k_1'+b_2'k_2'+h_2'\xi_{n_2',r})}\, x^{n_1'/2}\, H_{n_1'}\left(\frac{1}{2\sqrt{\left(\sin\frac{\pi x}{L}\right)^{b_1'}}}\right)\binom{n_1'+\alpha}{n_1'}$$

$$\left(\frac{\alpha+\beta+n'_2+1}{(\alpha+1)_{k'}}\right)\overline{\psi}(S_{h'_2,r})z_2^{S_{h'_2,r}}\frac{\left(\sin\frac{\pi\lambda_m}{L}\right)\left(\sin\frac{\pi\lambda_m ct}{L}\right)}{2\pi\lambda_m-\sin 2\pi\lambda_m}\sin\frac{\lambda_m\pi x}{L}$$

$$\overline{H}^{M'_1,N'_1+1}_{P'_1+1,Q'_1+1}\left[\left(\frac{z_1}{2^{h_1}}\right)\middle|\begin{matrix}(1-u'-b'_1k'_1-b'_2k'_2-h'_2\xi_{h'_2,r},h'_1;1),(a_j,\alpha_j;A_j)_{1,N'_1},(a_j,\alpha_j)_{N'_1+1,P'_1}\\(b_j,\beta_j)_{1,M'_1},(b_j,\beta_j;B_j)_{M'_1+1,Q'_1}.(1-u'-b'_1k'_1-b'_2k'_2-h'_2\xi_{h'_2,r}\mp\lambda_m,h'_1;1)\end{matrix}\right] \qquad (4.1)$$

The condition of validity of above result can easily obtain from the result (2.1).

5. REFERENCES

1. Bushman, R.G. and Shrivastava, H.M.; The $\overline{H}$ -function associated with a certain class of Feynman integral. J. Phys. A: Math. Gen. 23, (1990), 4707-4710.
2. Gradshteyn, I.S. and Ryzhik, I.M.; Tables of Integrals, Series and Products, Academic Press, Inc. New York, (1980).
3. Inayat-Hussain,; A.A. New Properties of Hypergeometric series derivable from Feynman integrals:II A generalization of the H-function, J. Phys. A: Math. Gen. 20 (1987), 4119-4128.
4. Powers, David, L.; Elementry Differential equations with boundary value problems, Weber and Schmidt, Boston.
5. Rathie, A.K.; A new generalization of generalized hypergeometric function, Le Matematiche Fasc II, 52 (1997), 297-310.
6. Srivastava, H.M.; A contour integral involving Fox's H-function, Indian J. Math. 14 (1972), 1-6.
7. Szegö,G.; Orthogonal polynomials, Amer. Math. Soc.Colloq.Publ. Vol.23 (1975), Fifth edition, Amer. Math. Soc. Providence, Rhode Island.

S.C.Sharma
Deparment of Mathematics,
University of Rajasthan,
Jaipur-302055, India
Email : suresh chand 26@gmail.com

Vandana Agrawal
Vivekananda College of Engineering .
VIT Campus,
Jaipur-303012, India
Email : r_bhargava72@yahoo.com

Received January 2011